# Laboratory investigation of significant gas transfer enhancement via capillary-gravity bow waves

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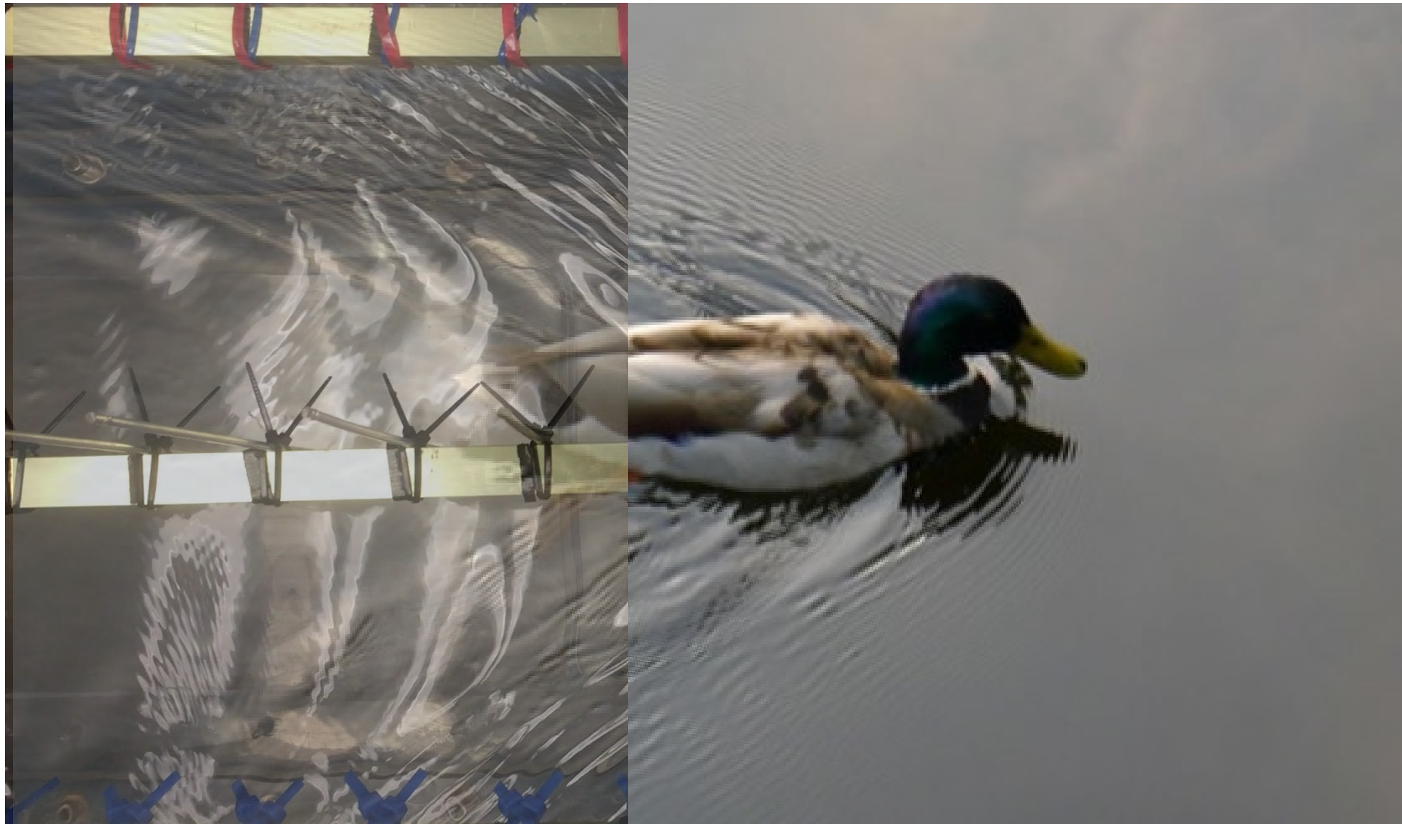

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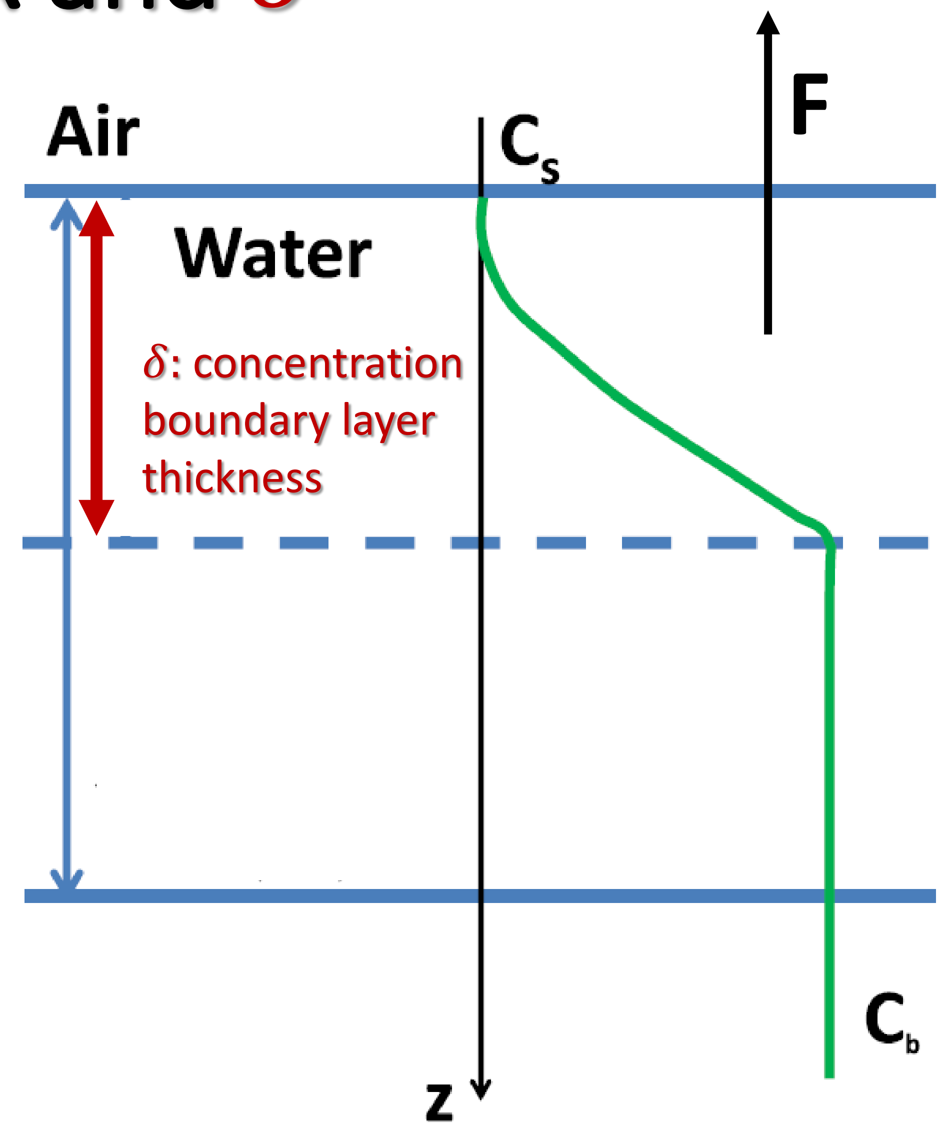
# Using standard definition of *k* and $\delta$

$$F = \boldsymbol{k}\Delta C = \boldsymbol{k}\left(\frac{C_a}{H} - C_b\right) = \boldsymbol{k}(C_s - C_b)$$

$F$ = mass flux/unit surface area (M $L^{-2}$ $T^{-1}$)
***k* = transfer velocity (L $T^{-1}$)**
$\Delta C$ = concentration gradient over CBL (M $L^{-4}$)
$C_a$ = the bulk air concentration (M $L^{-3}$)
$C_b$ = the bulk water concentration (M $L^{-3}$)
$H$ = Henry's Law constant

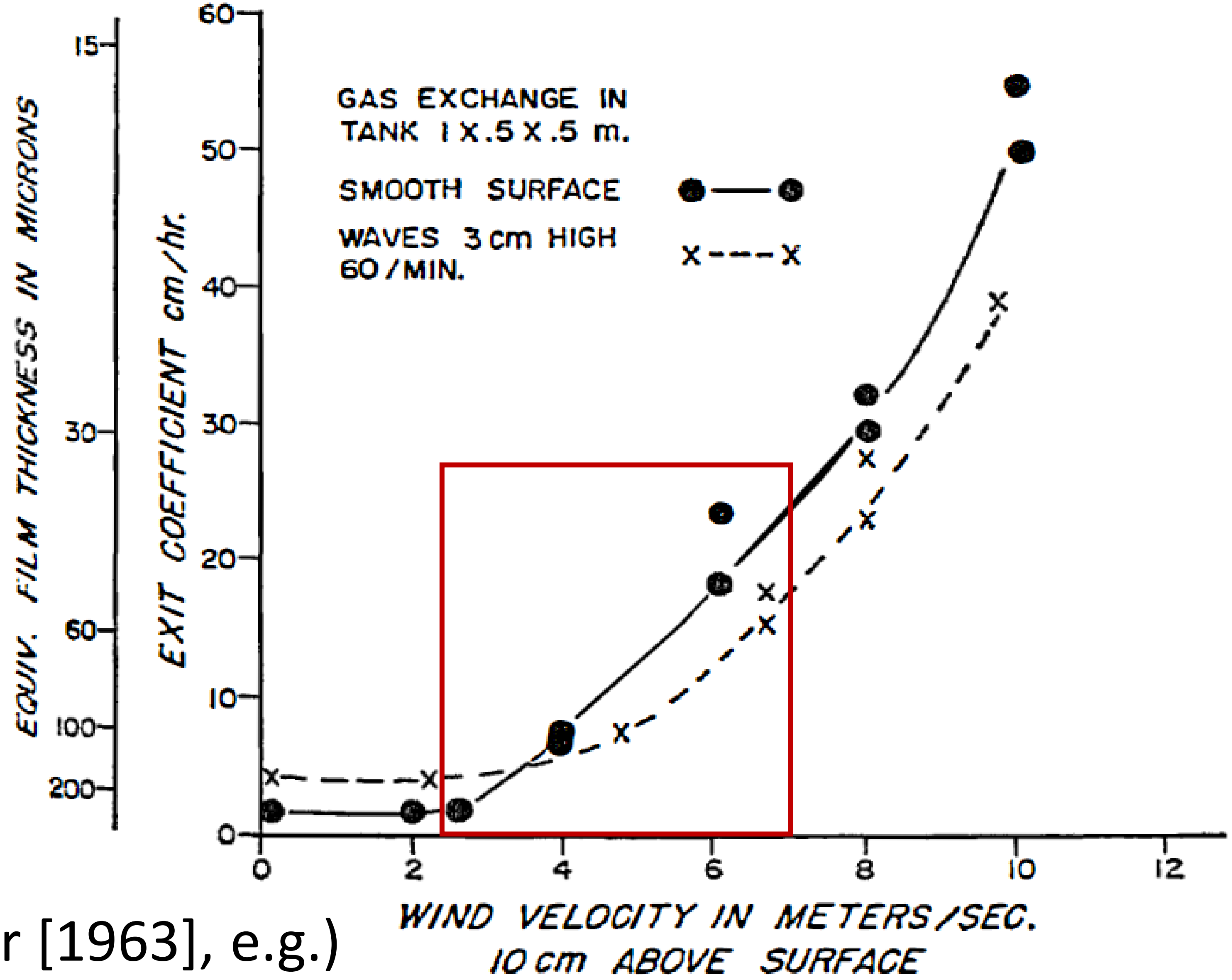



(Nermst, 1904; Figure modified from Katul and Liu, 2017)

# $k$ is enhanced at onset of wind waves

For $U_{10} < 7\,\frac{m}{s}$ capillary waves dominate

(e.g., Bourasa *et al.*, 1999)



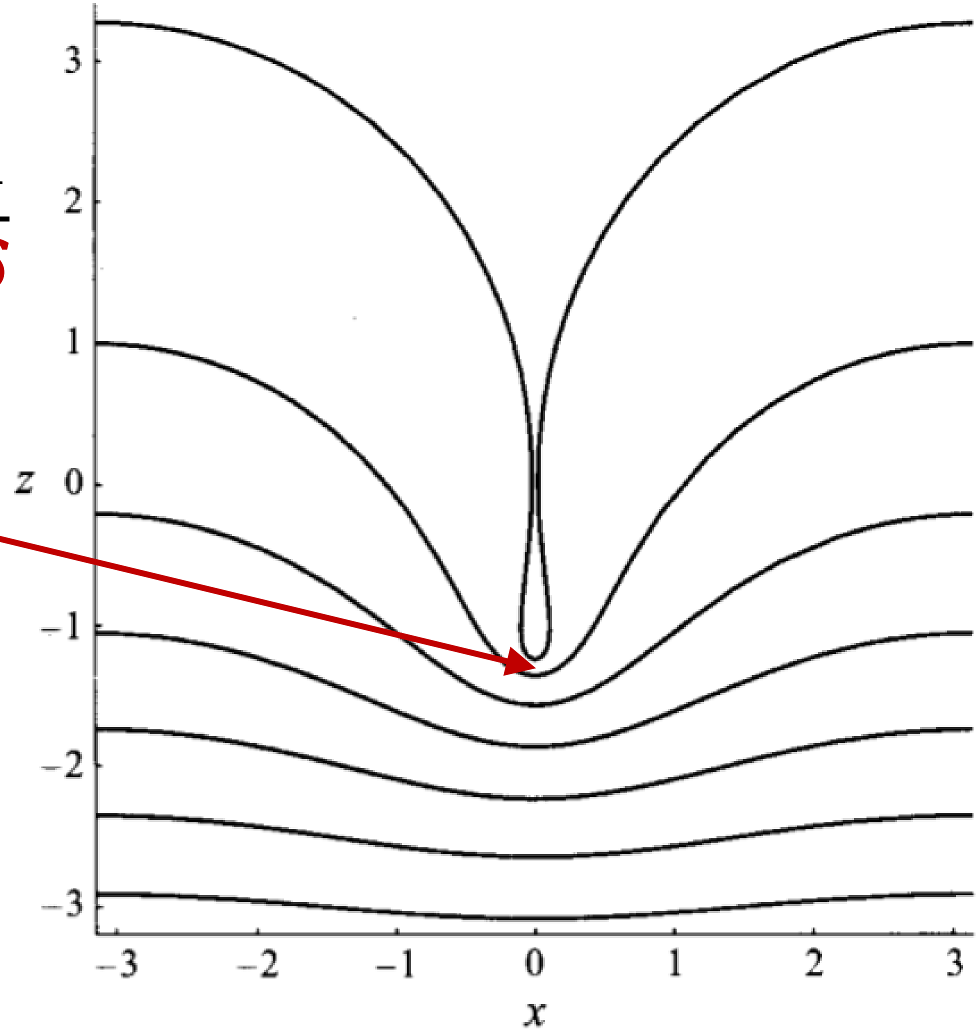

(Figure from Kanwisher [1963], e.g.)

# Capillary waves thin $\boldsymbol{\delta}$, increase concentration gradient

Fick's Law:

$$F = D\frac{\partial C}{\partial z} \rightarrow k \propto \frac{1}{\boldsymbol{\delta}}$$

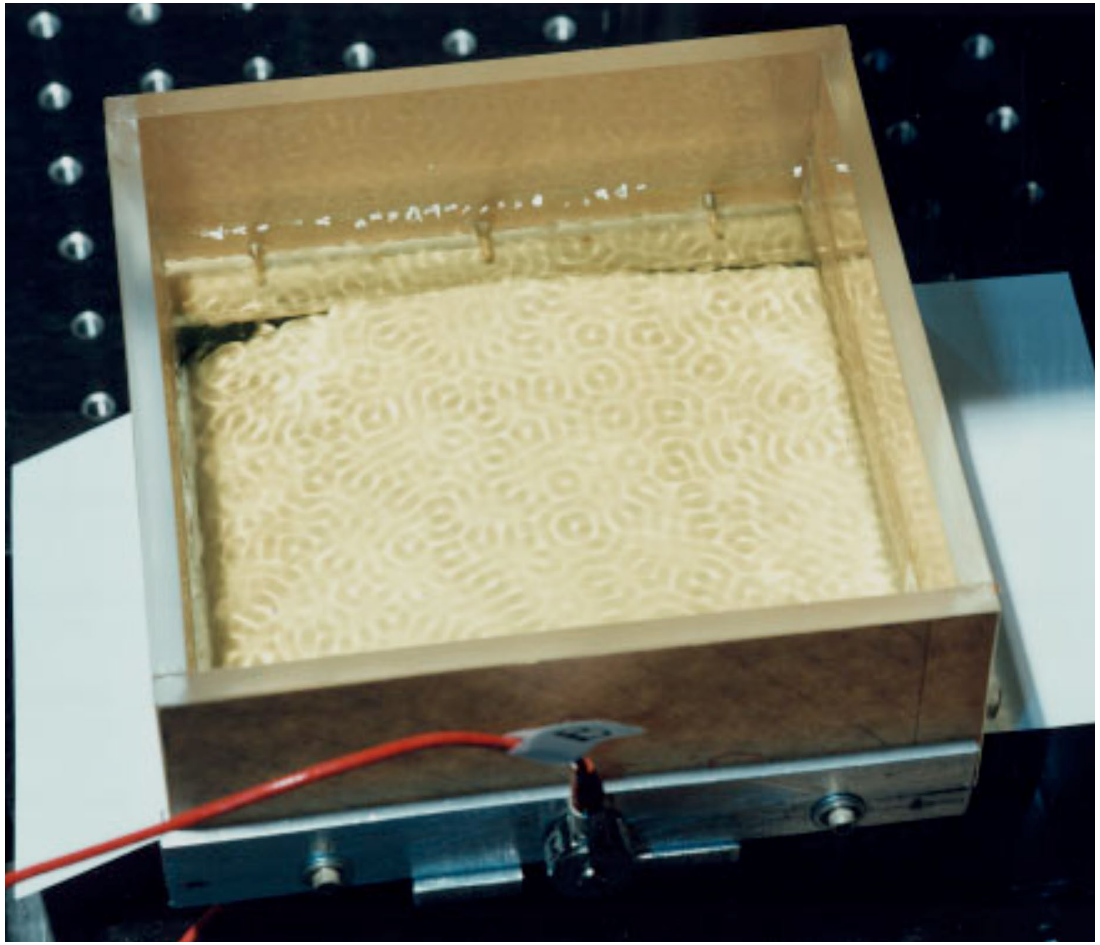
Straining troughs leads to thinner $\boldsymbol{\delta}$

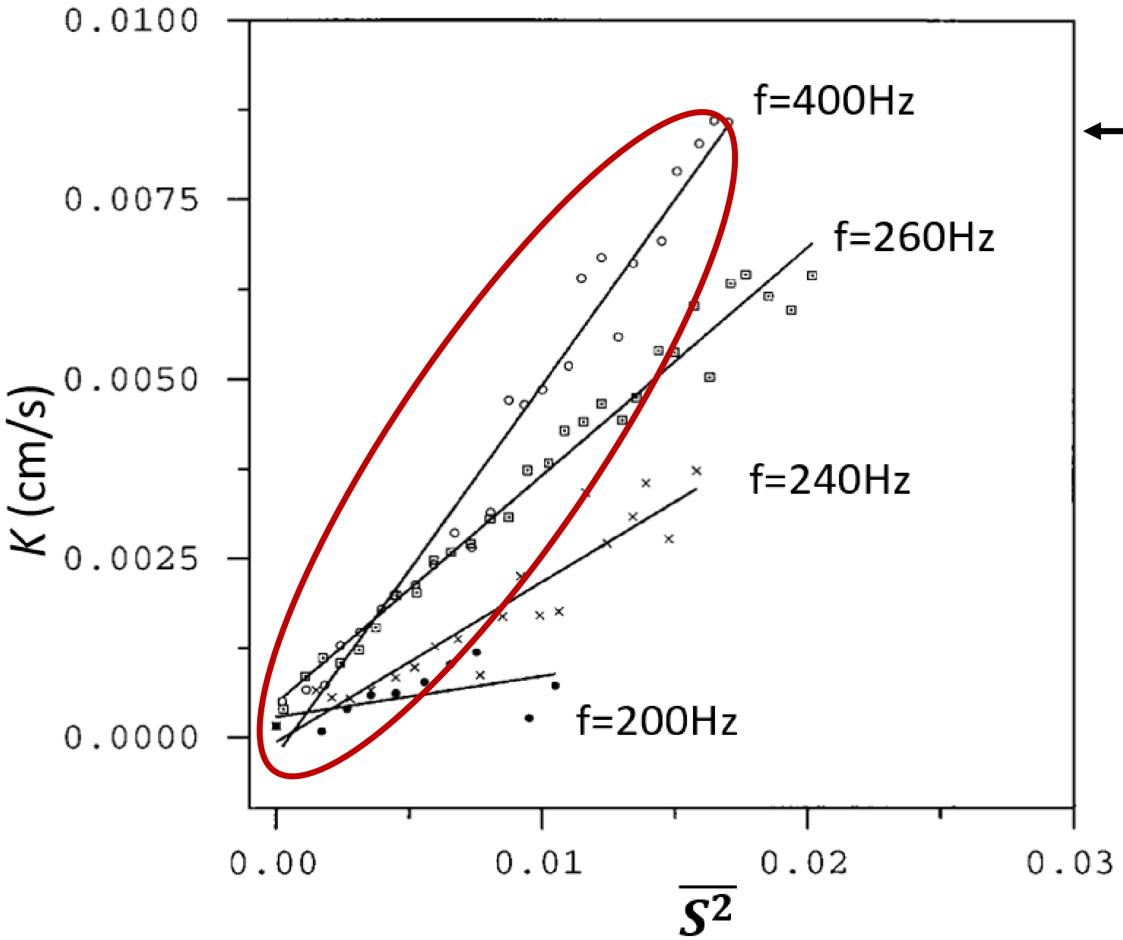



Szeri model estimates **3-fold** enhancements

The surface elevation and streamlines of the exact solution for a capillary wave amplitude A = 0.45 (Szeri, 1997).

# Small-scale model → almost 100x increase in *k*






Maximum *k*
~31.5 cm/h

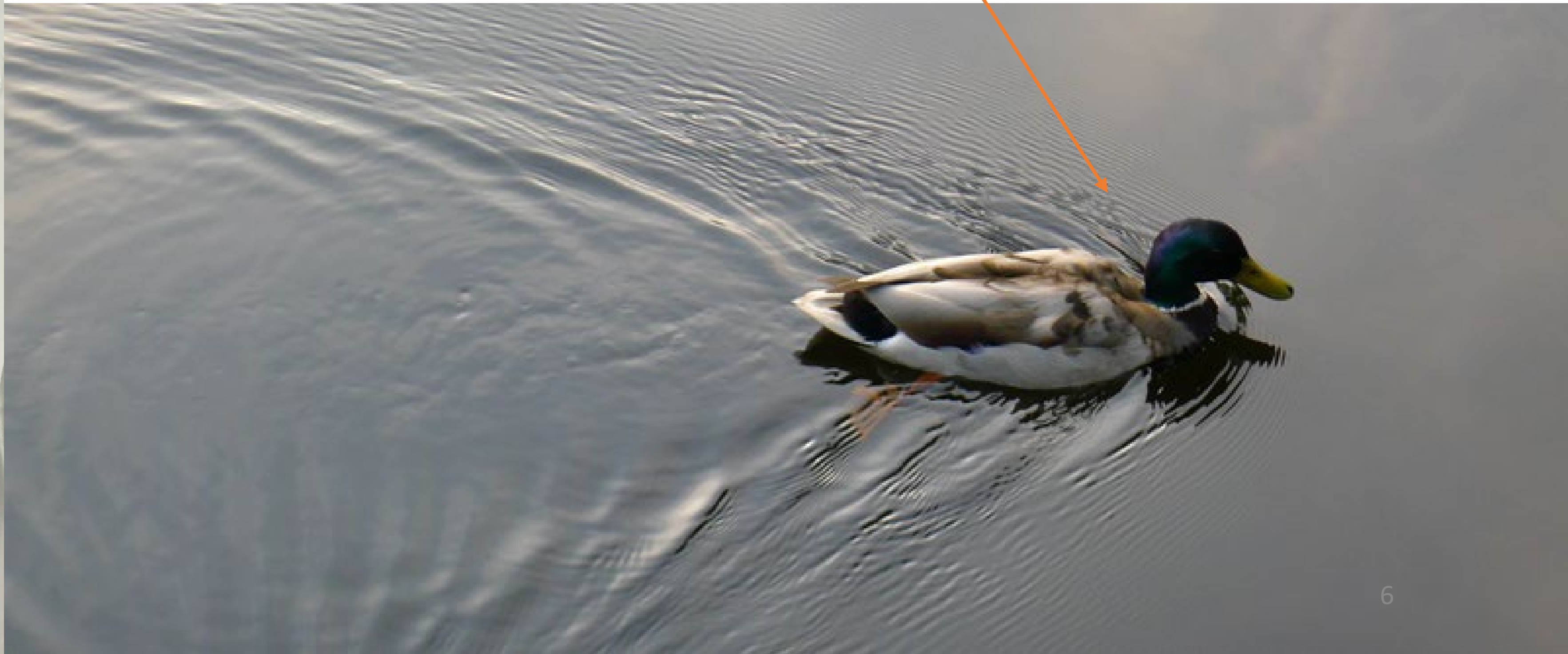
*S* = the slope of the water surface

$2.26\ mm \leq \lambda \leq 3.62\ mm$

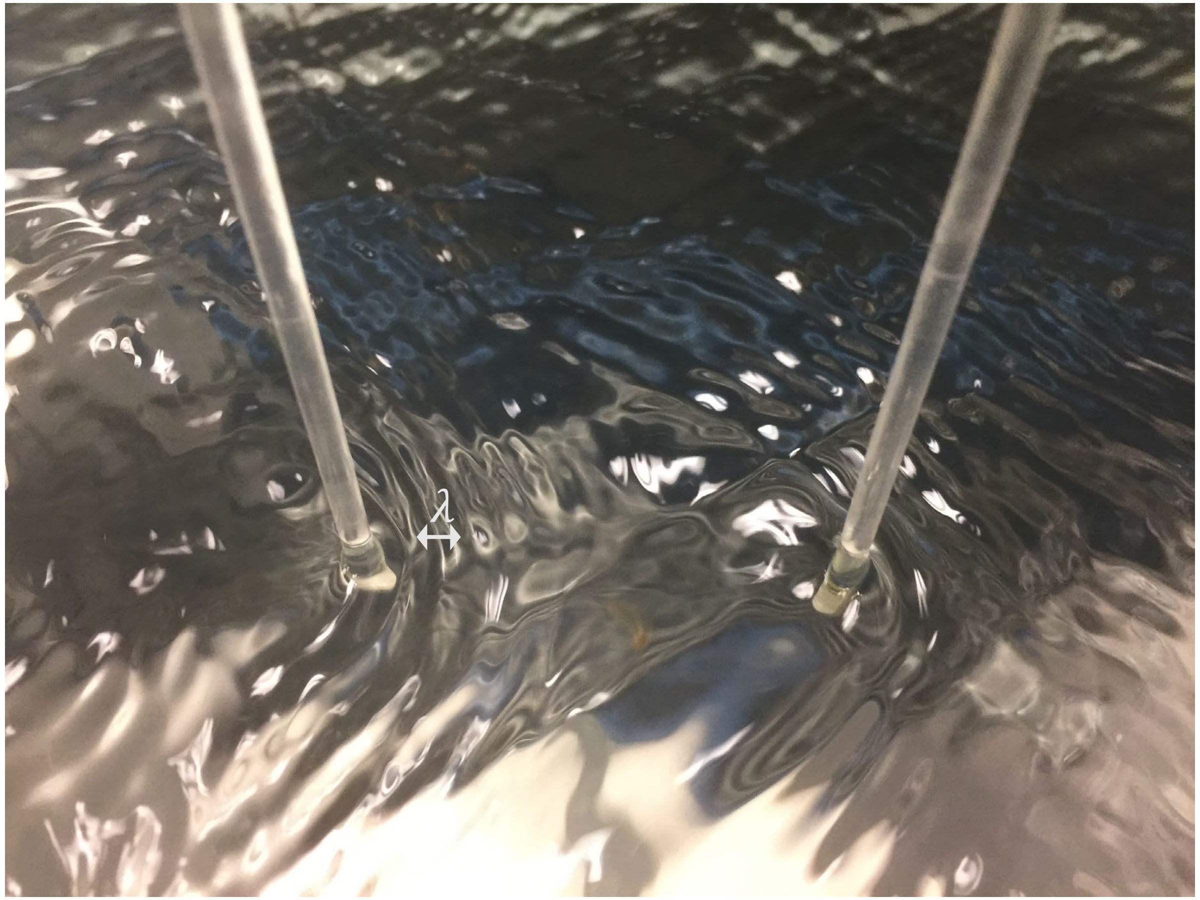
(Saylor & Handler, 1999)

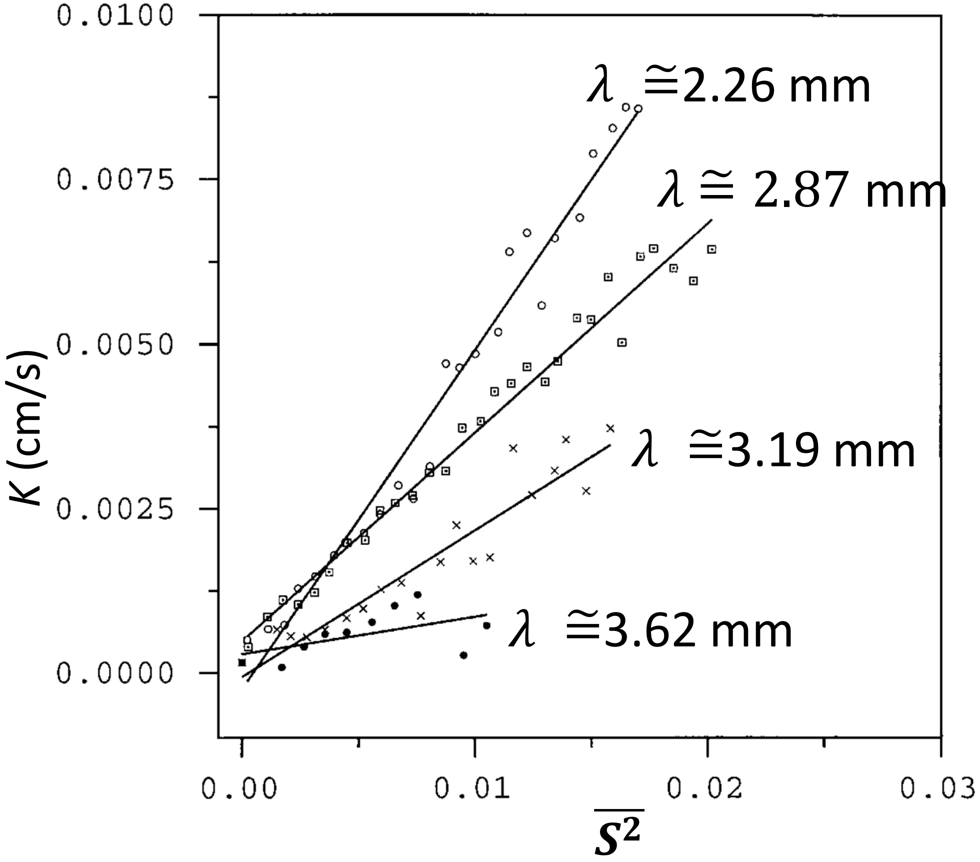
# We expand to a larger scale by generating capillary-gravity bow waves

Capillary-gravity bow waves (mm-cm-scale)



# Greatest *k* enhancement at shortest wavelength

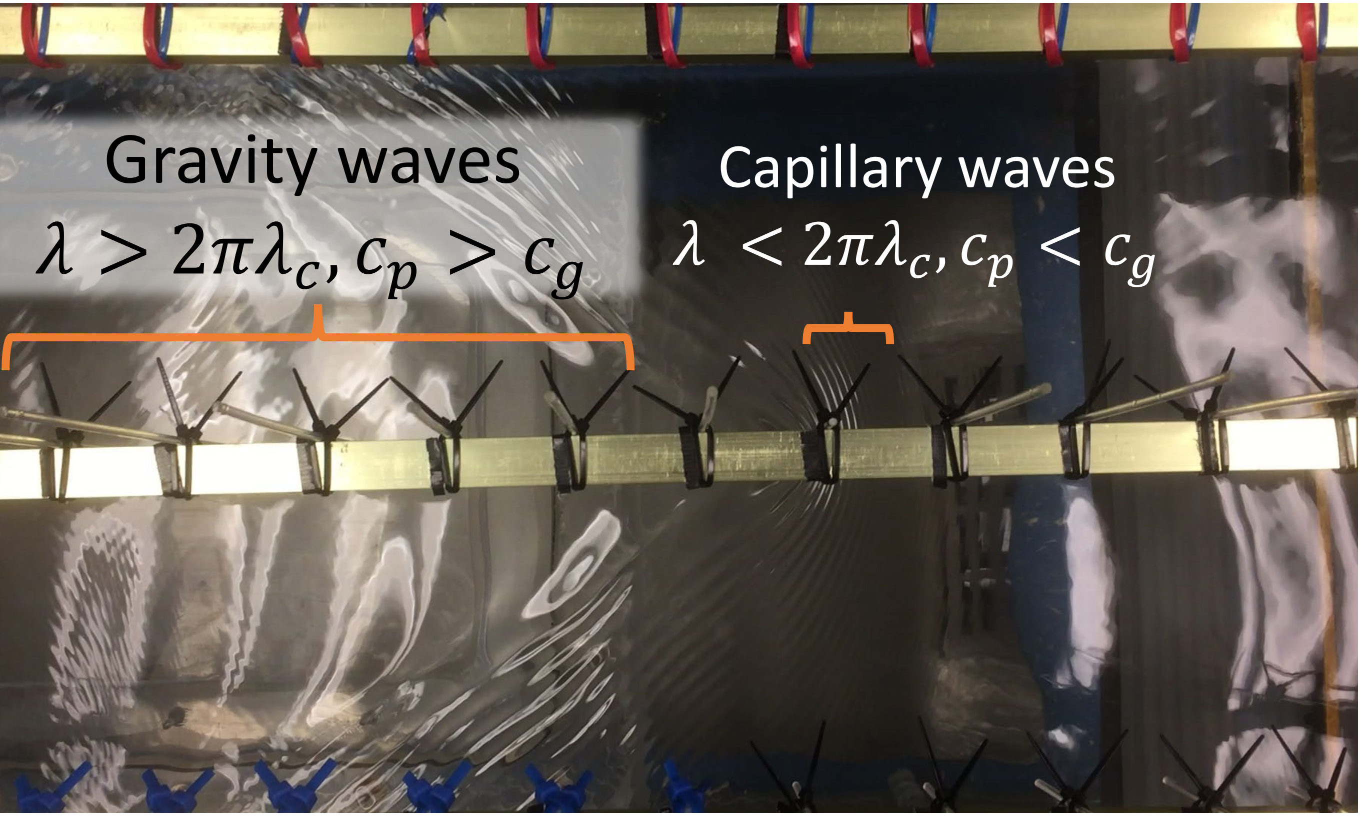





(Saylor & Handler, 1999)

# Analytical model to target relative velocities




← Mean flow

$$\lambda_c = \sqrt{\frac{\Upsilon}{\rho g}}$$ ← Surface tension

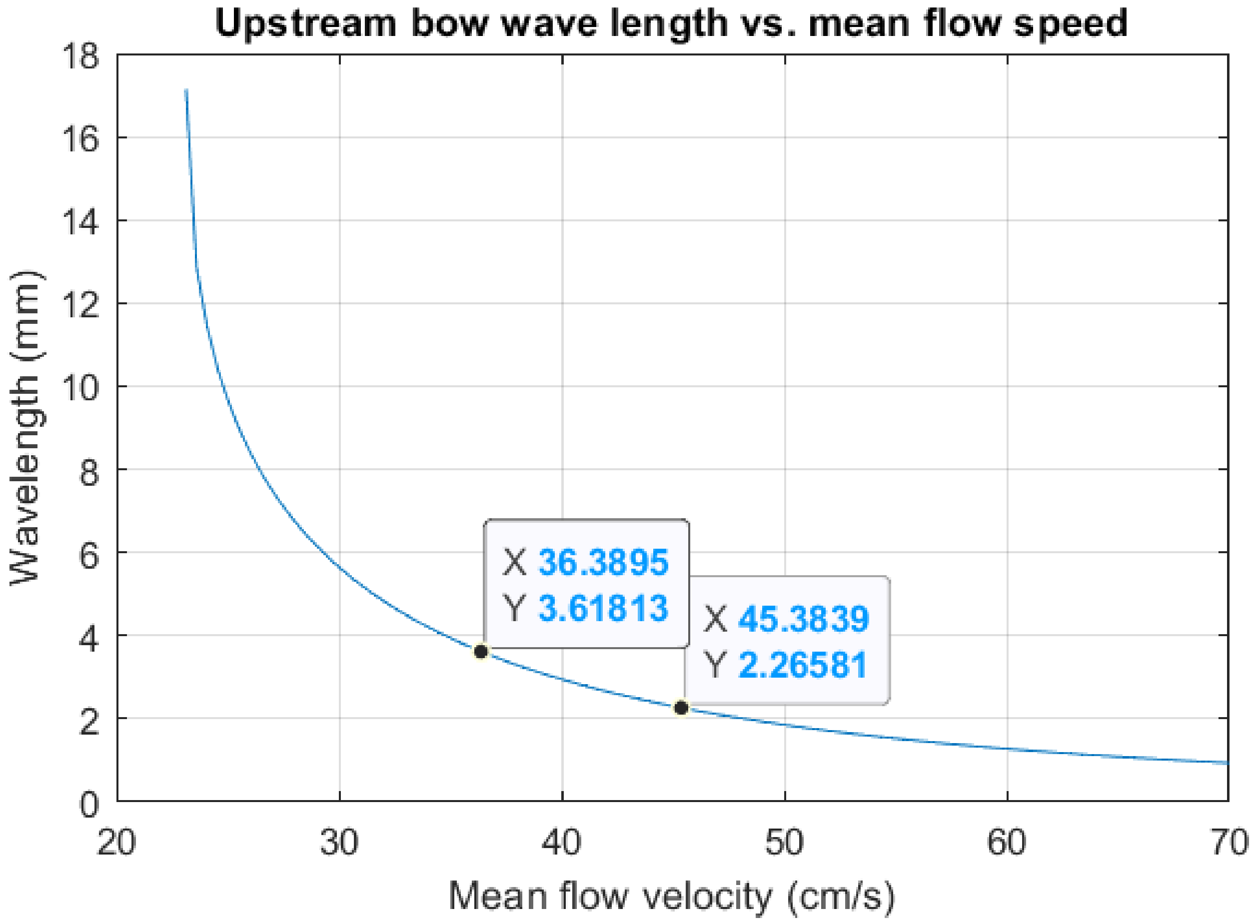
$$c_p = \sqrt{\frac{g}{K} + \frac{\Upsilon K}{\rho}}$$ ← Wave number

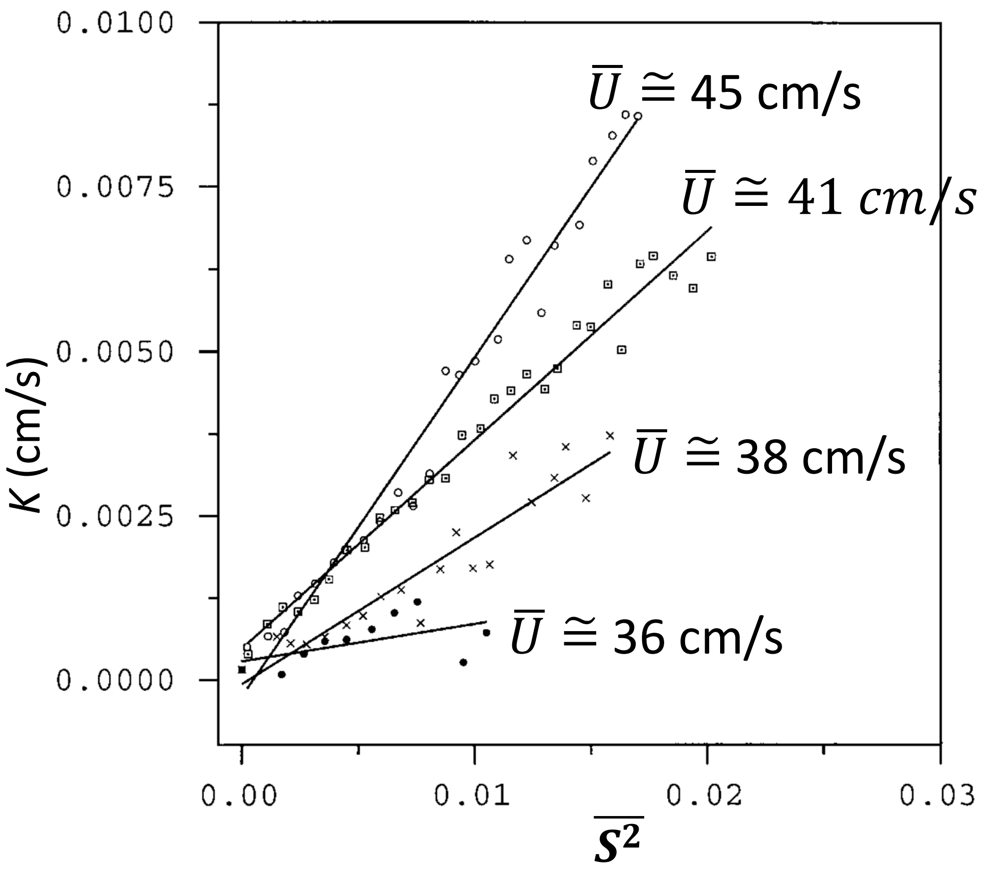
$$c_g = \frac{g + \frac{3\Upsilon K^2}{\rho}}{2\sqrt{gK + \frac{\Upsilon K^3}{\rho}}}$$

$$K_{C/G} = \frac{1}{\lambda_c} U^2 \sqrt{\frac{\rho}{4g\Upsilon}} \left\{ 1 \pm \sqrt{1 - \frac{4g\Upsilon}{\rho U^4}} \right\}$$

(Raphaël and De Gennes, 1995)

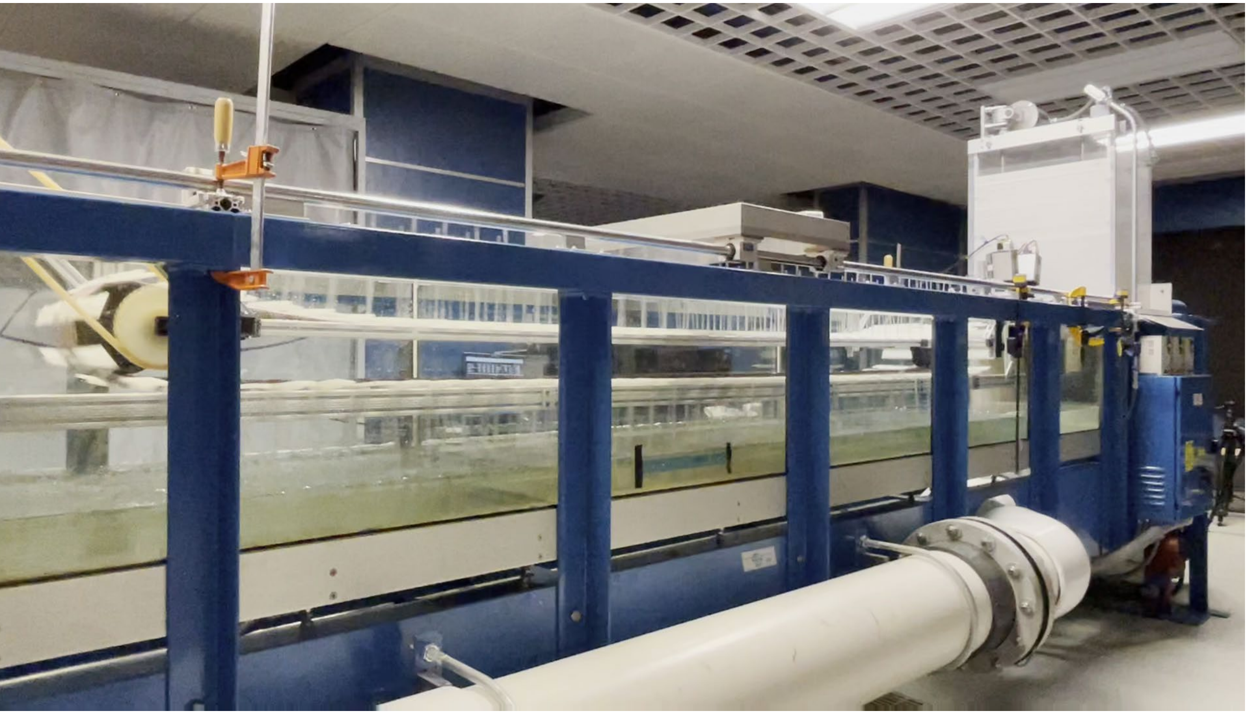
# Target velocities around 36-45 cm/s and higher



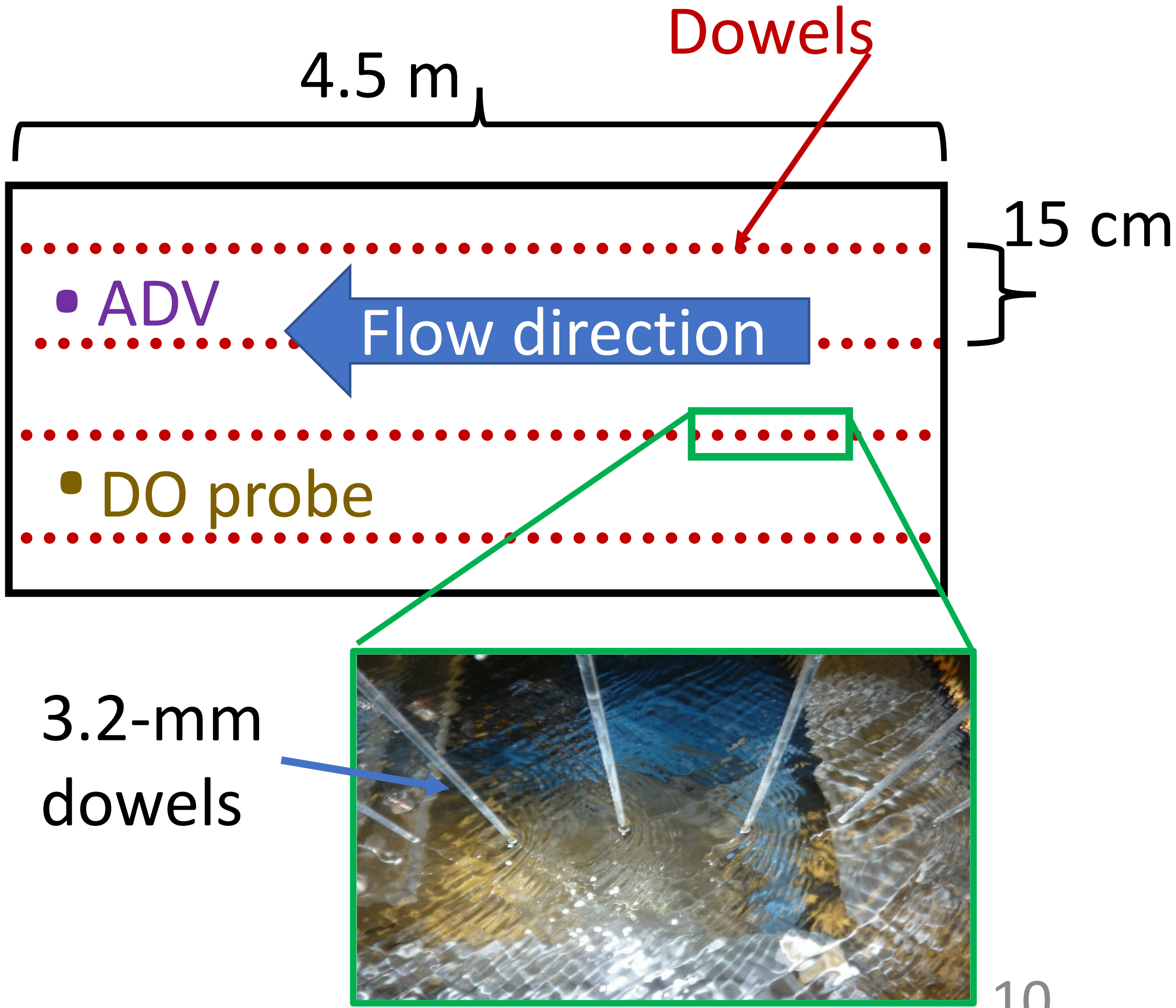

(Right figure modified from Saylor & Handler, 1999)

# Dowel array to populate entire surface



Recirculating flume with suspended dowel array

**Top view**



# Remove dissolved oxygen and measure reaeration

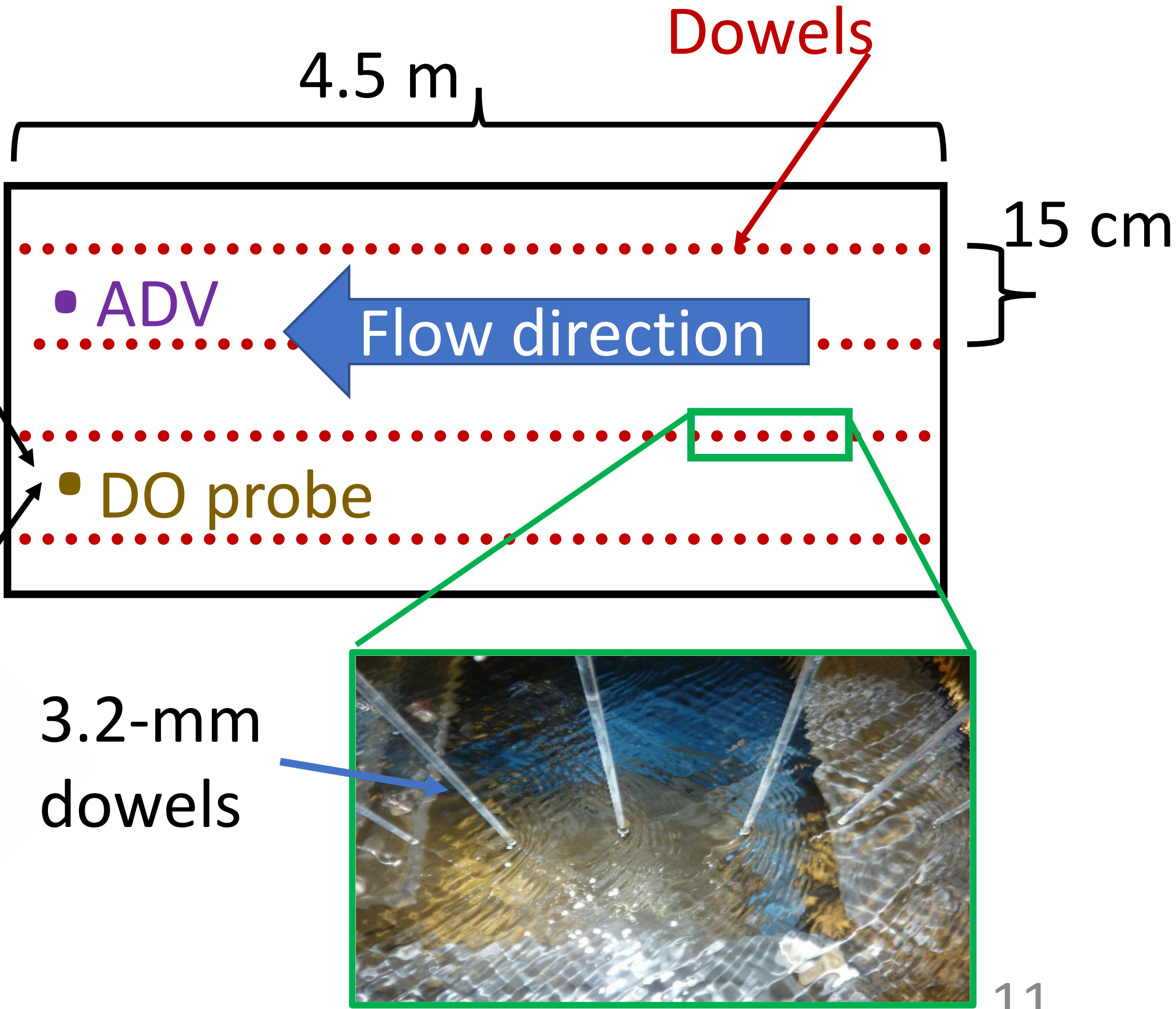
Steps:

- Use $CoCl_2$ and $Na_2SO_3$ to remove oxygen
- Measure voltage, temperature, pressure over time
- Use temperature-sensitive calibration to convert voltage to concentration
- Solve for theoretical saturation concentration

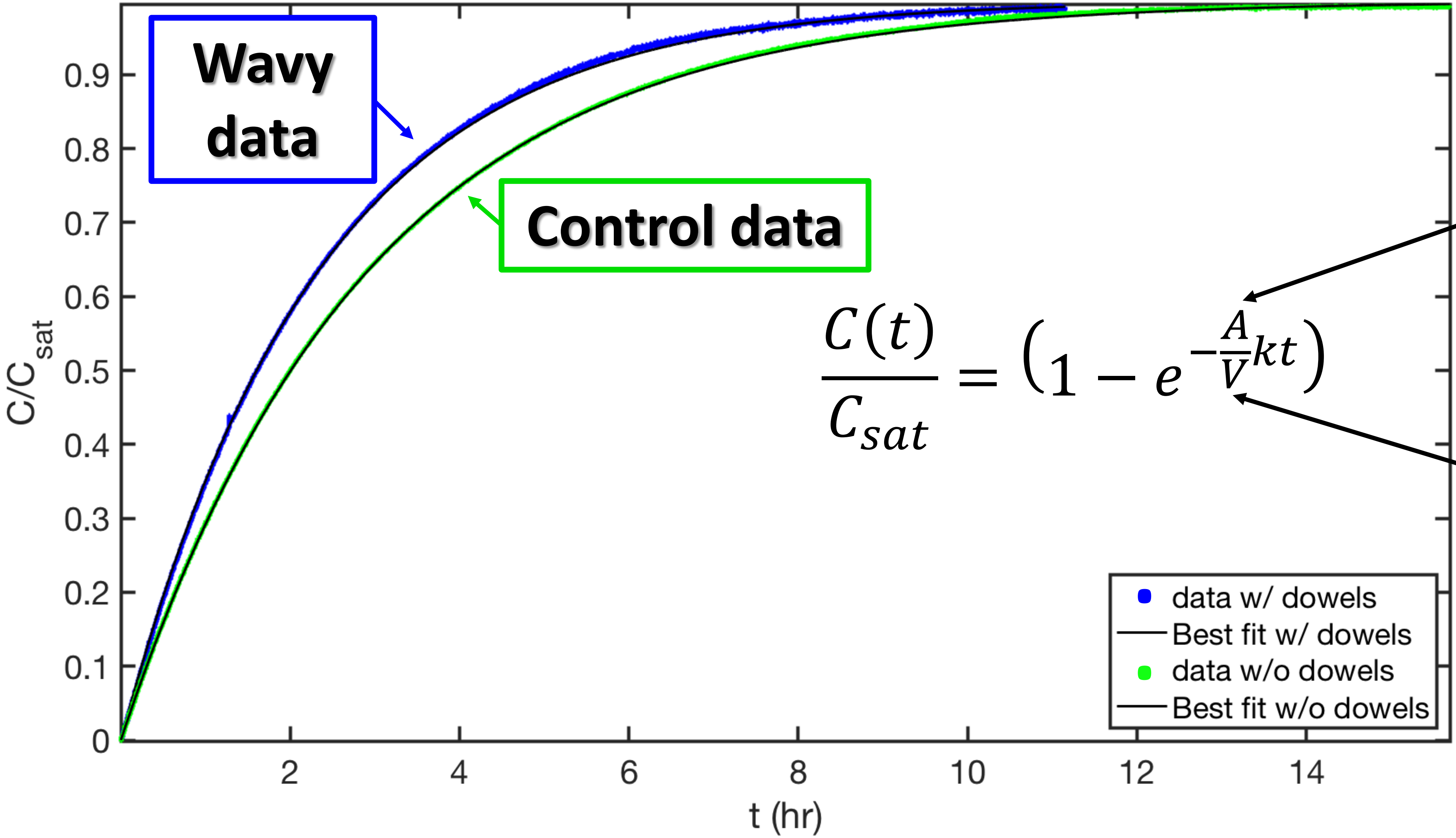
YSI DO Probe


PME miniDOT

**Top view**



# *k* is calculated from DO measurements

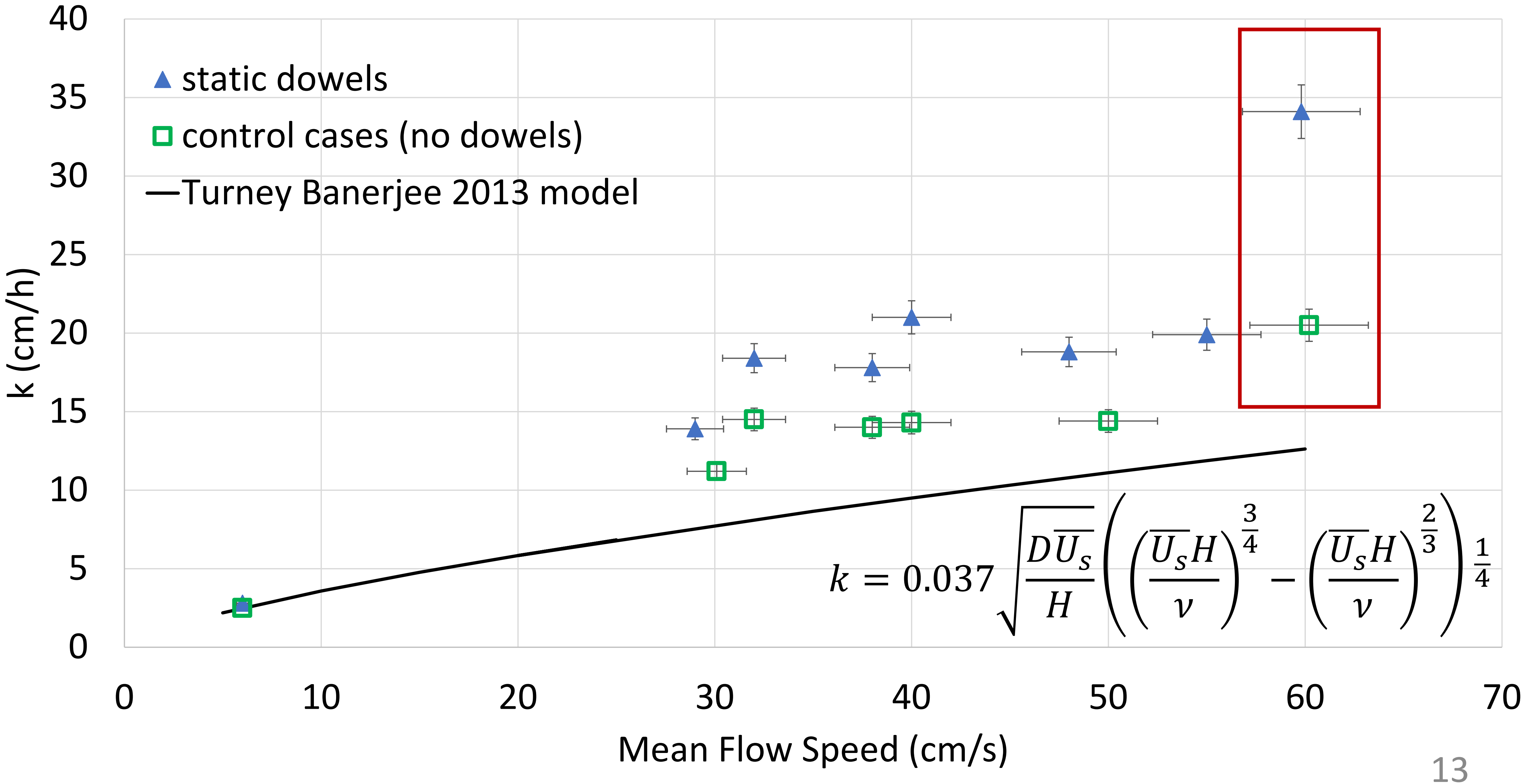



$$\frac{C(t)}{C_{sat}} = \left(1 - e^{-\frac{A}{V}kt}\right)$$

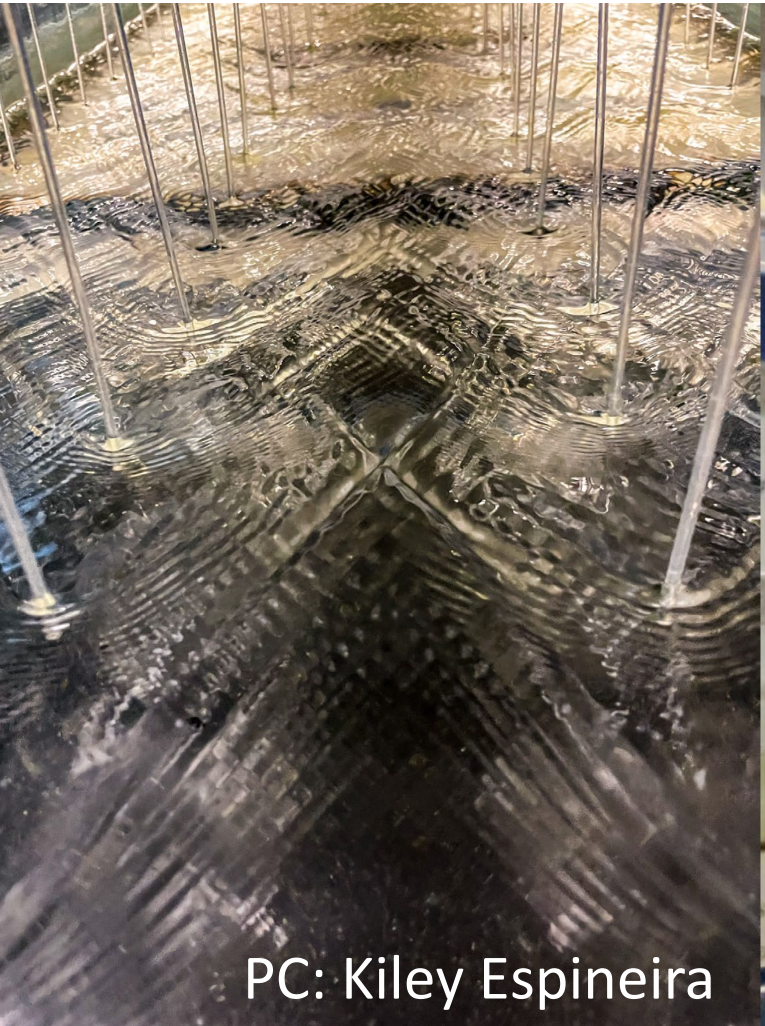
Area

Volume

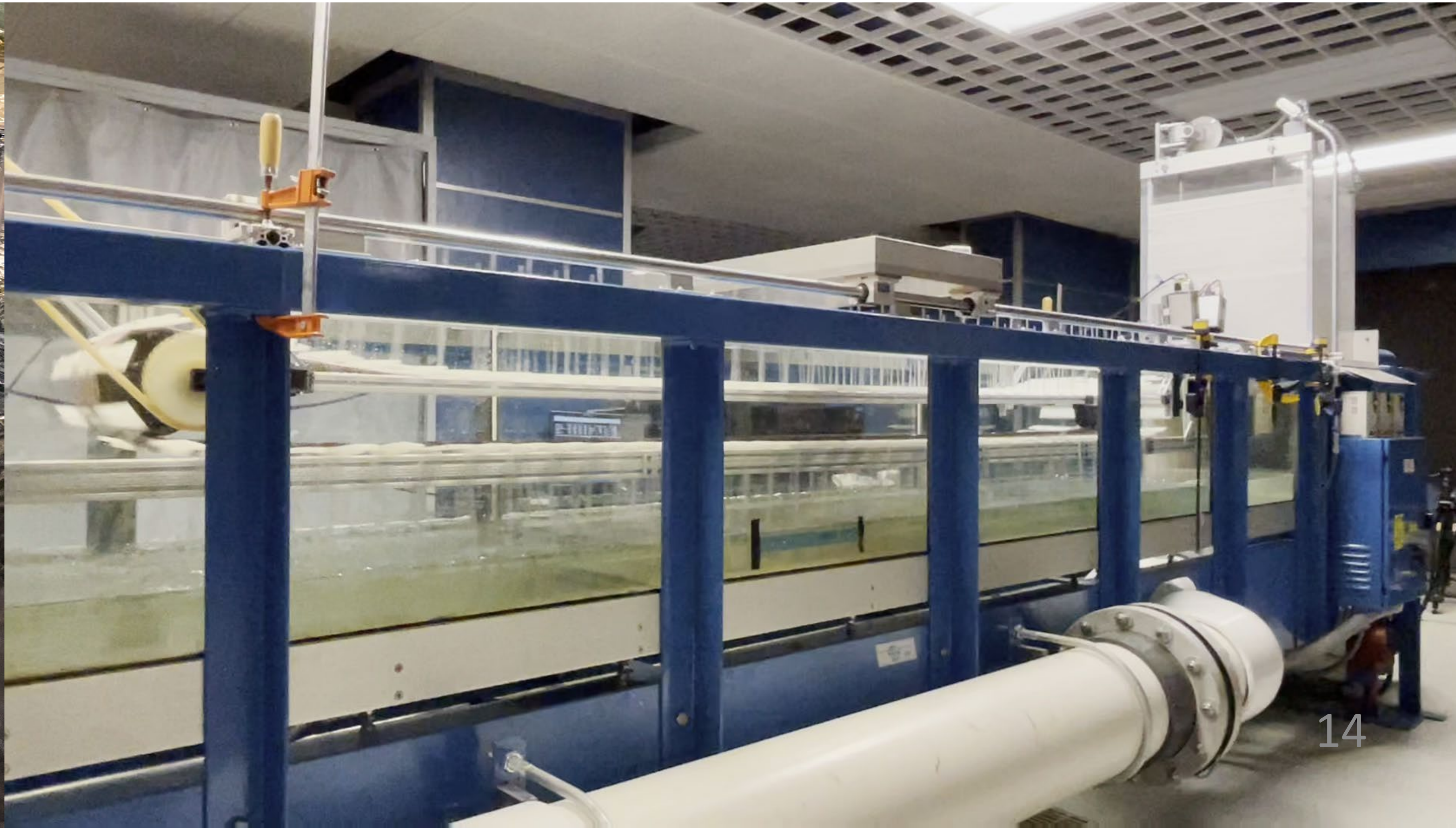
# Results with static dowels: up to **66%** enhancement



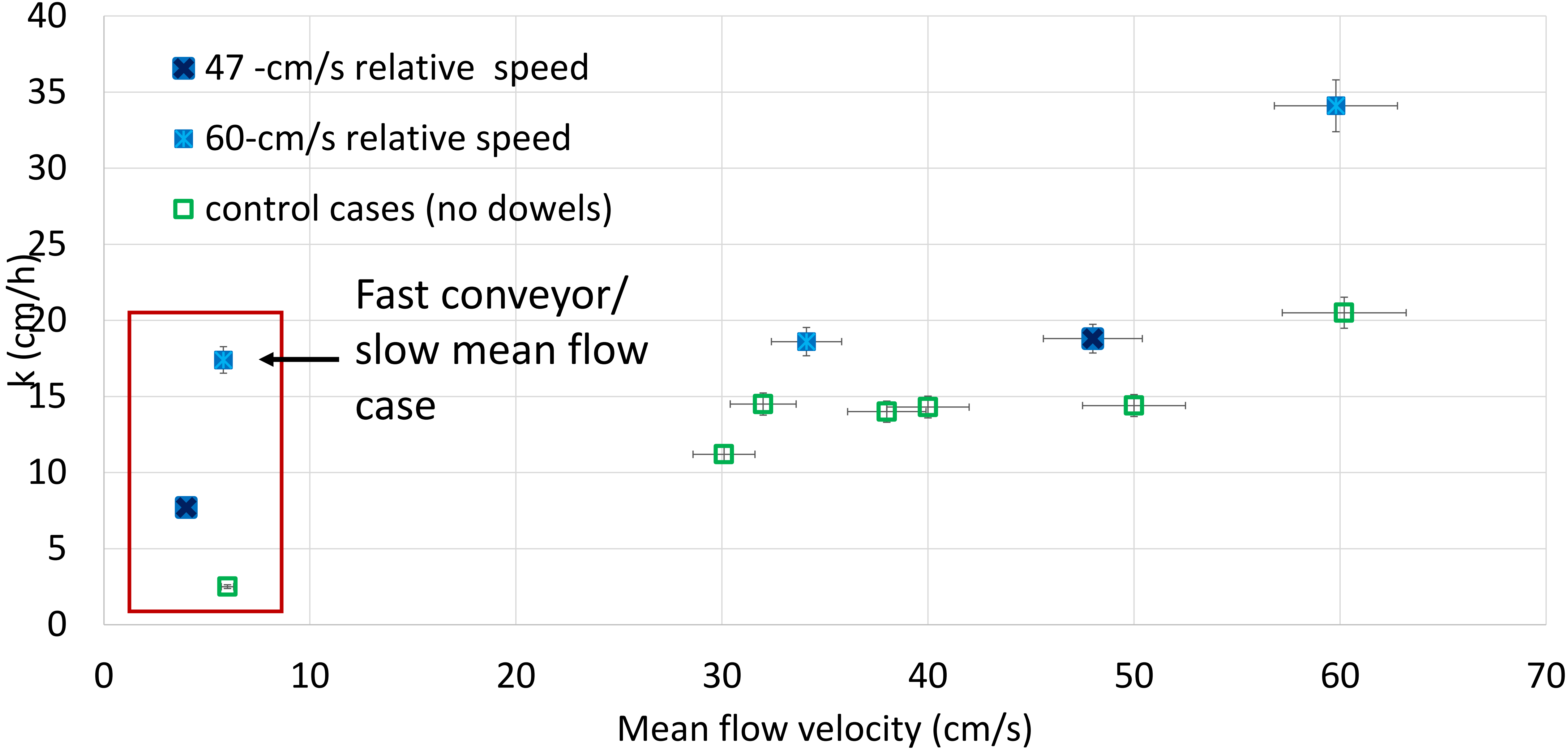
# Conveyor belt isolates waves from background turbulence and BL dynamics



PC: Kiley Espineira



# Moving dowels: Almost 6-fold increase in slow mean flow case

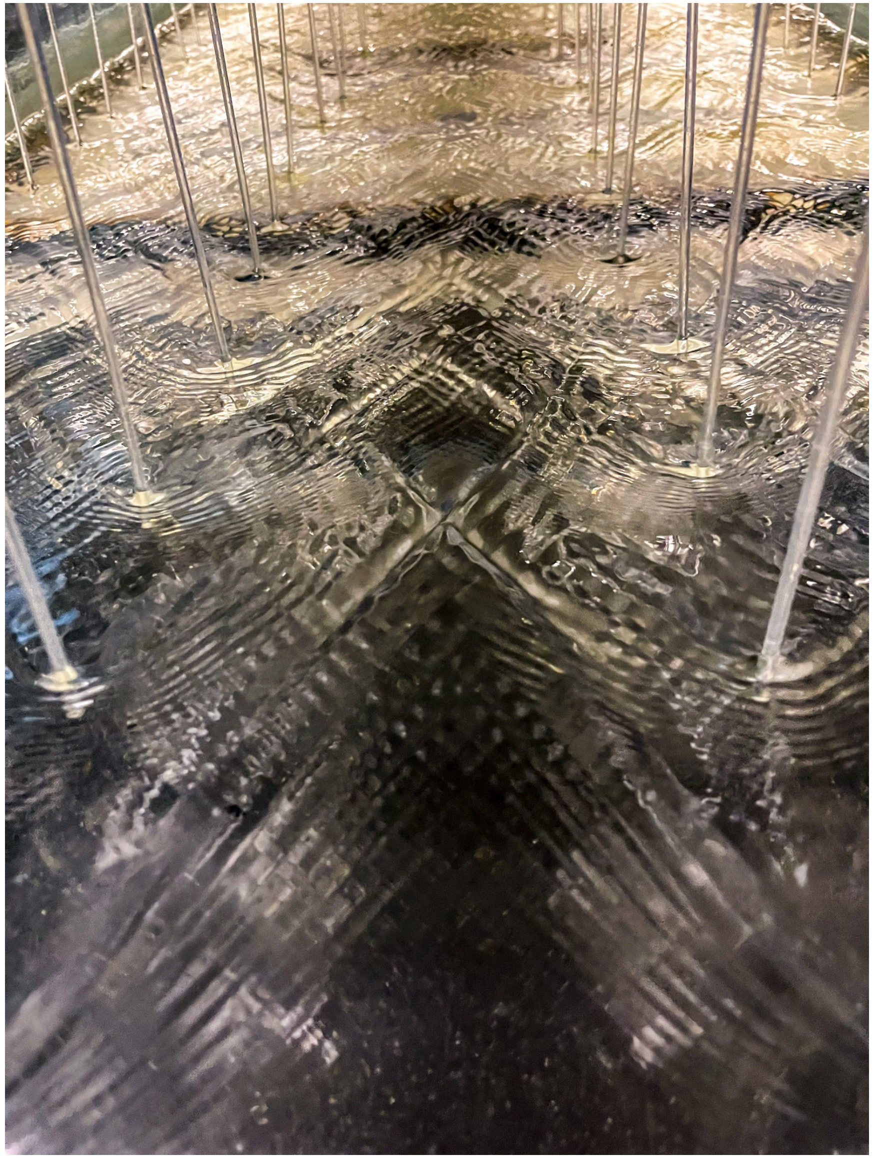


# Summary

- Measured up to 67% enhancement in *k* due to static dowels at 60-cm/s flow speed.
- Measured a greater than 5-fold increase in *k* due to dowels conveyed at 60 cm/s against 5-cm/s flow speed.

# Future Work

- Measure mean surface slope
- Use our dataset to develop a model that predicts these enhancements more broadly



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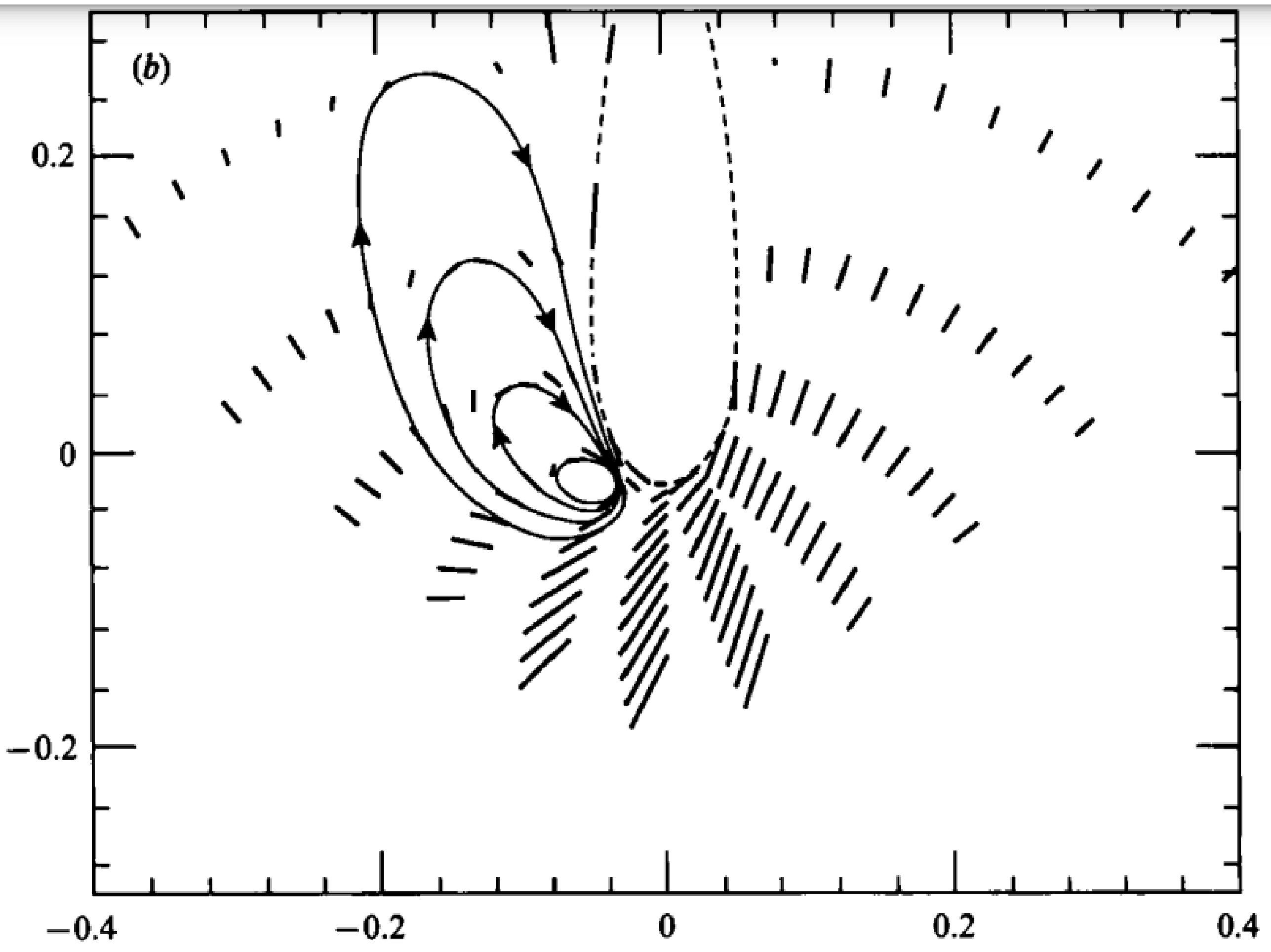
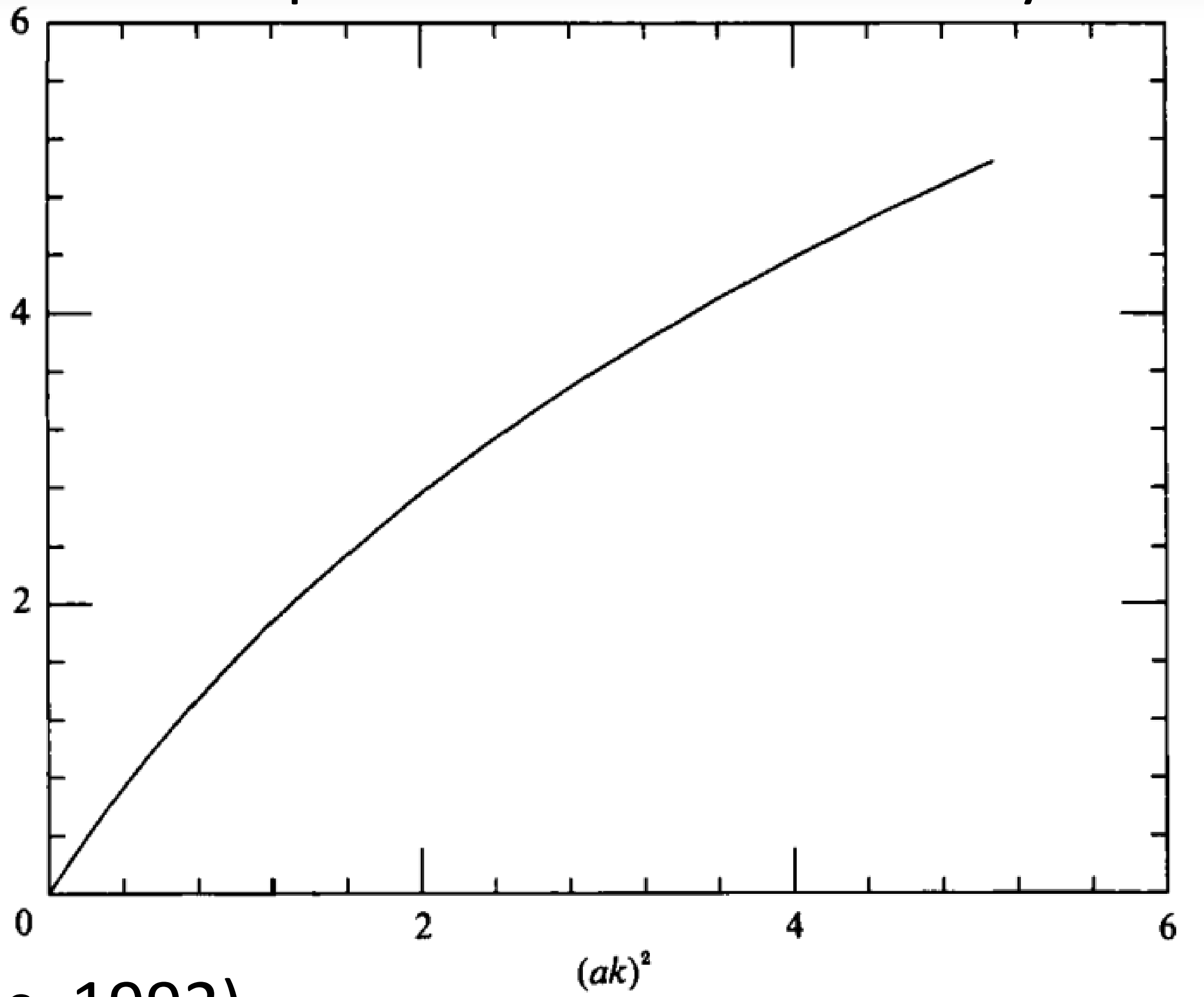
# Appendix

# Capillary-gravity waves increase vorticity:




Steeper waves=more vorticity



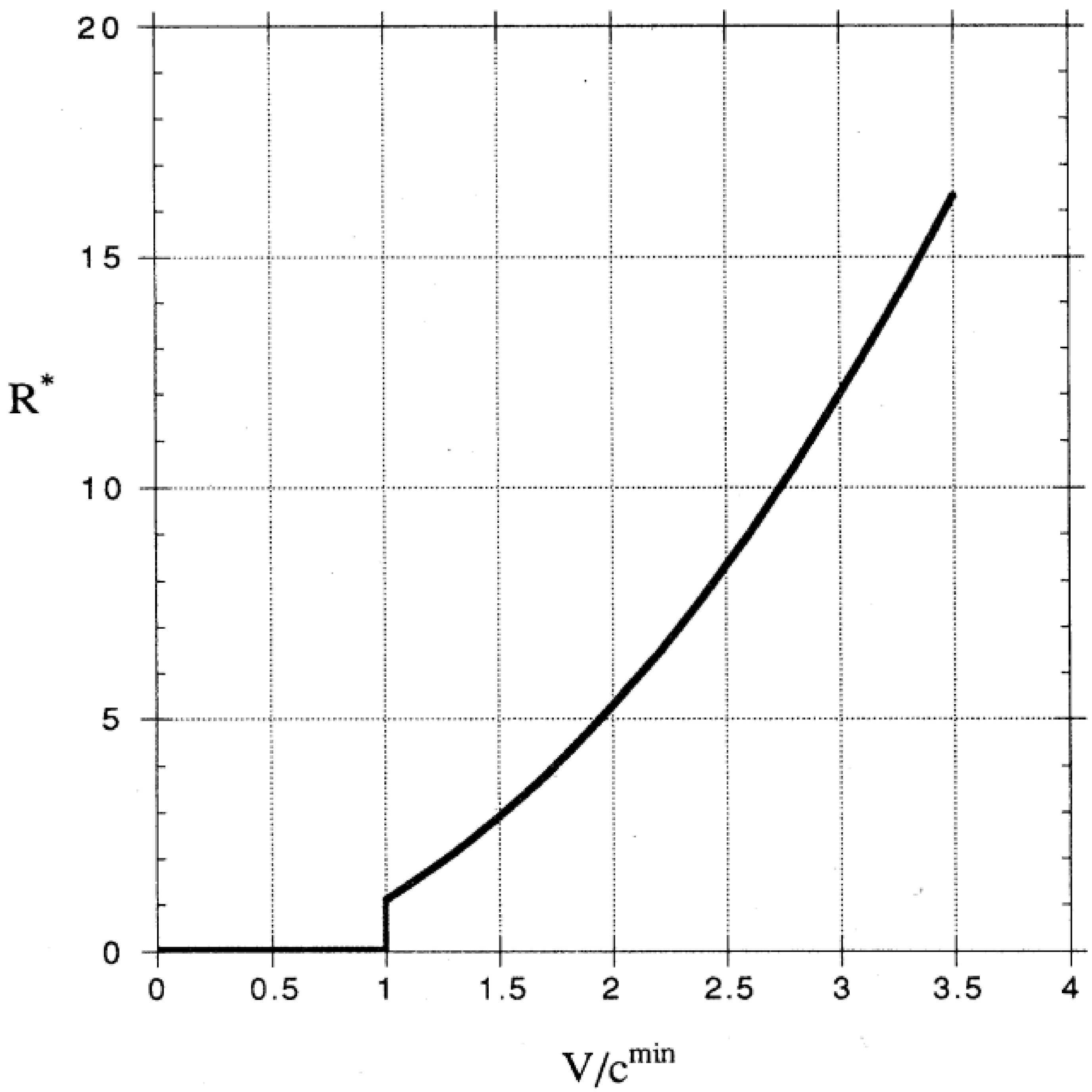

(Longuet-Higgins, 1992)

Moving dirac delta pressure field
(Raphael & De Gennes, 1996)

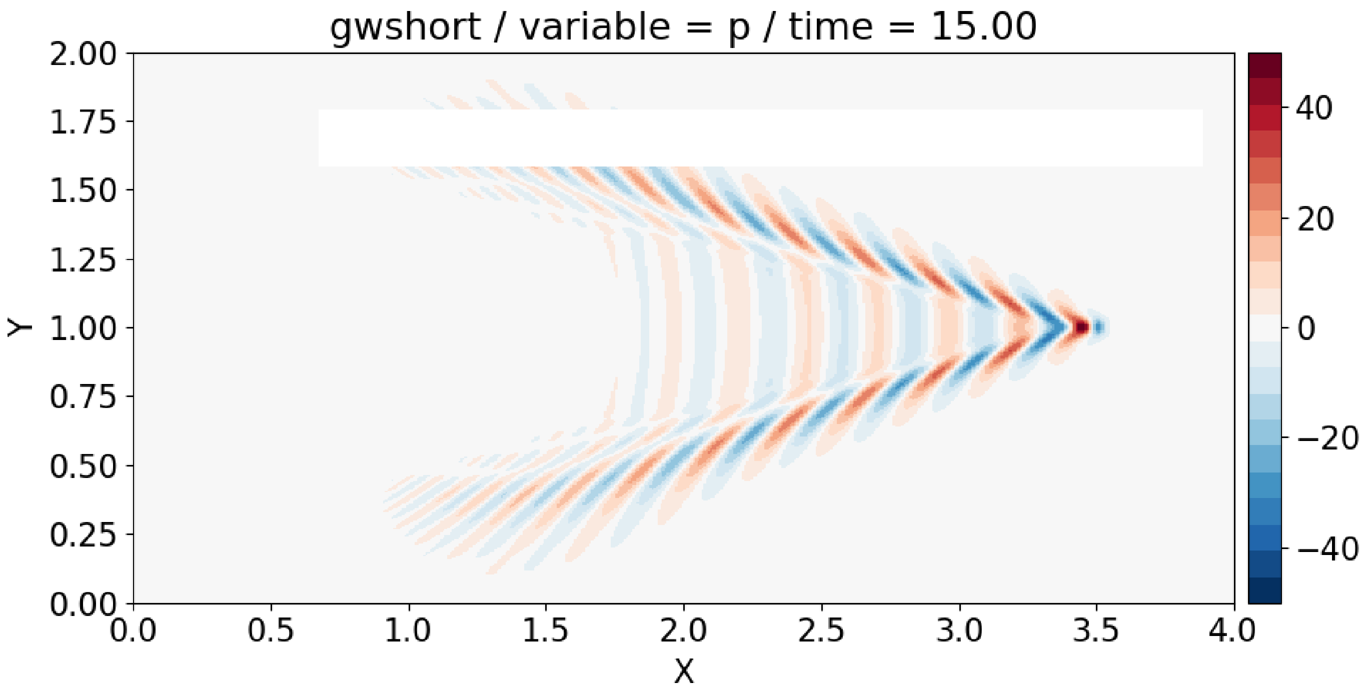
$$R^* = \pi\gamma(p^2\kappa)^{-1} \; R$$

$$R = \frac{2}{3}\frac{p^2\rho V^2}{\pi\gamma^2}, \quad V \gg c^{\min}$$

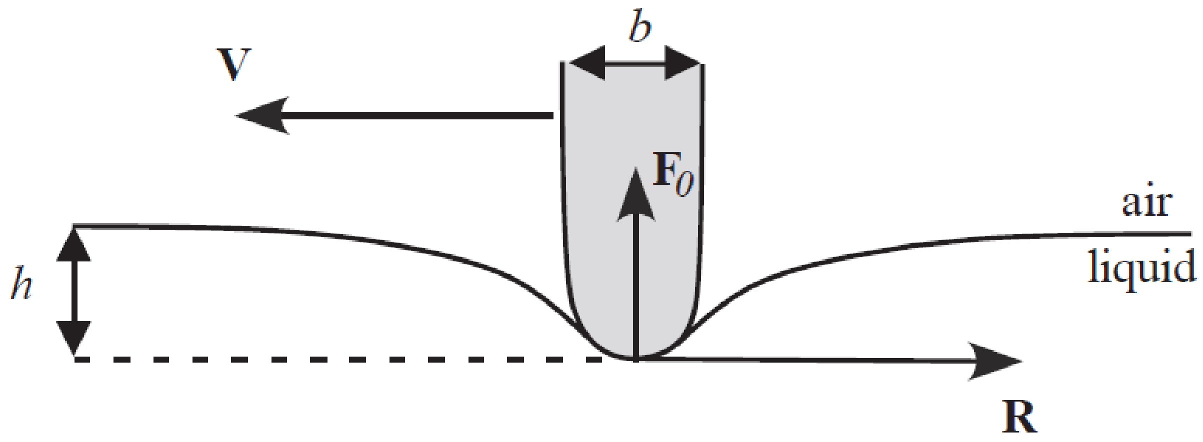
$$\kappa = (\rho g/\gamma)^{1/2}$$



# Cannot analytically relate to amplitude yet




Dr. Guillaume Roullet's wave2D model for gravity bow waves




(Chevy and Raphael, 2001)

**Amplitude**

$$A(\mathbf{K}_x, \mathbf{K}_y) = \frac{-ik_x V\, \widehat{P}(\mathbf{K}_x, \mathbf{K}_y)}{\Upsilon k(\mathbf{K}^2 + \mathbf{K}^2_{min}) - \rho V^2 \mathbf{K}_x^2}$$

$F_0 = f(V)$

**FT of Pressure field**

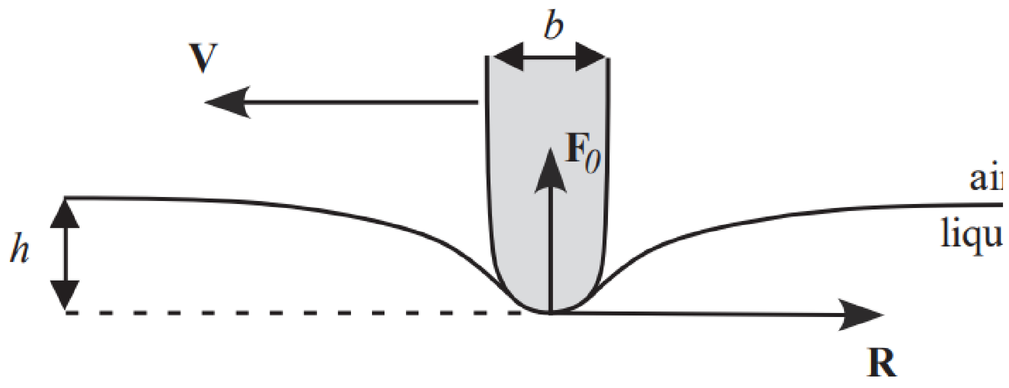
$$\widehat{P}(\mathbf{K}_x, \mathbf{K}_y) = F_0 \widehat{\phi}(K_x, K_y)$$

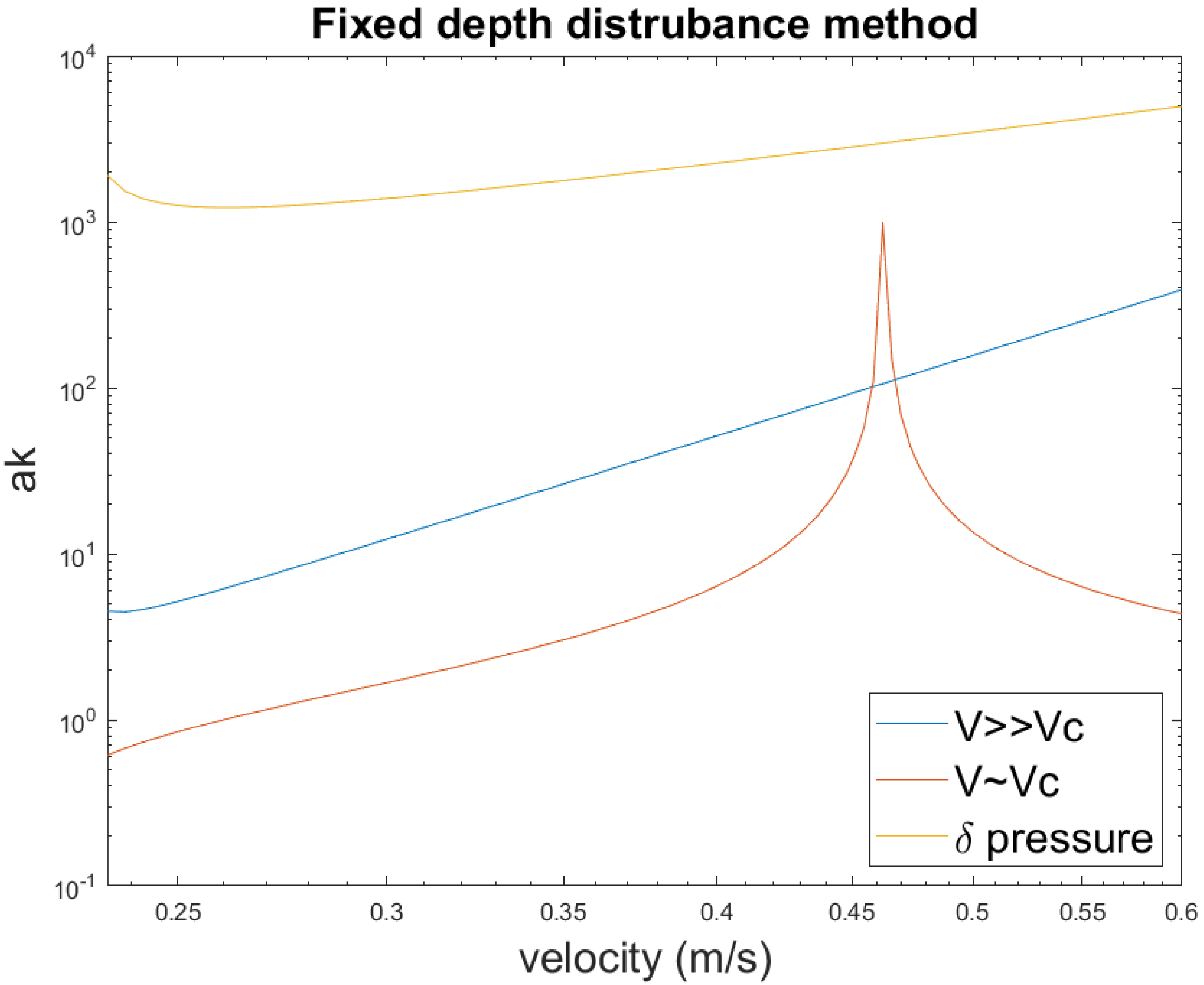
(in Rayleigh's linearized framework)

$$\hat{\xi}(K_x, K_y) = \frac{\frac{-F_0 K}{\rho}\widehat{\phi}(K_x, K_y)}{\omega^2 - 4\nu^2 K^3 q + (2\nu K^2 - iV \cdot K)^2}$$

**FT of free surface displacement**

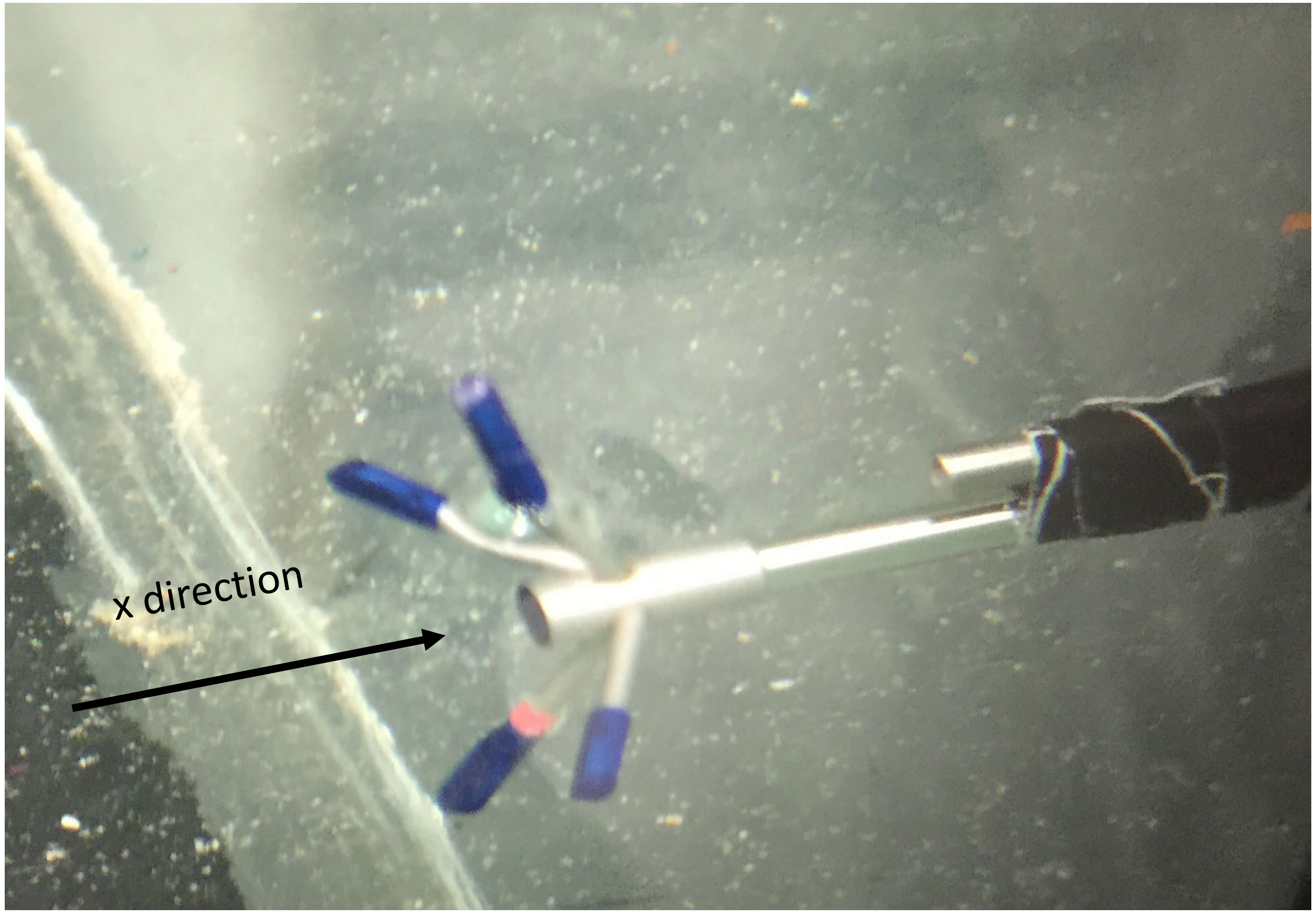
# Relating steepness and flow velocity

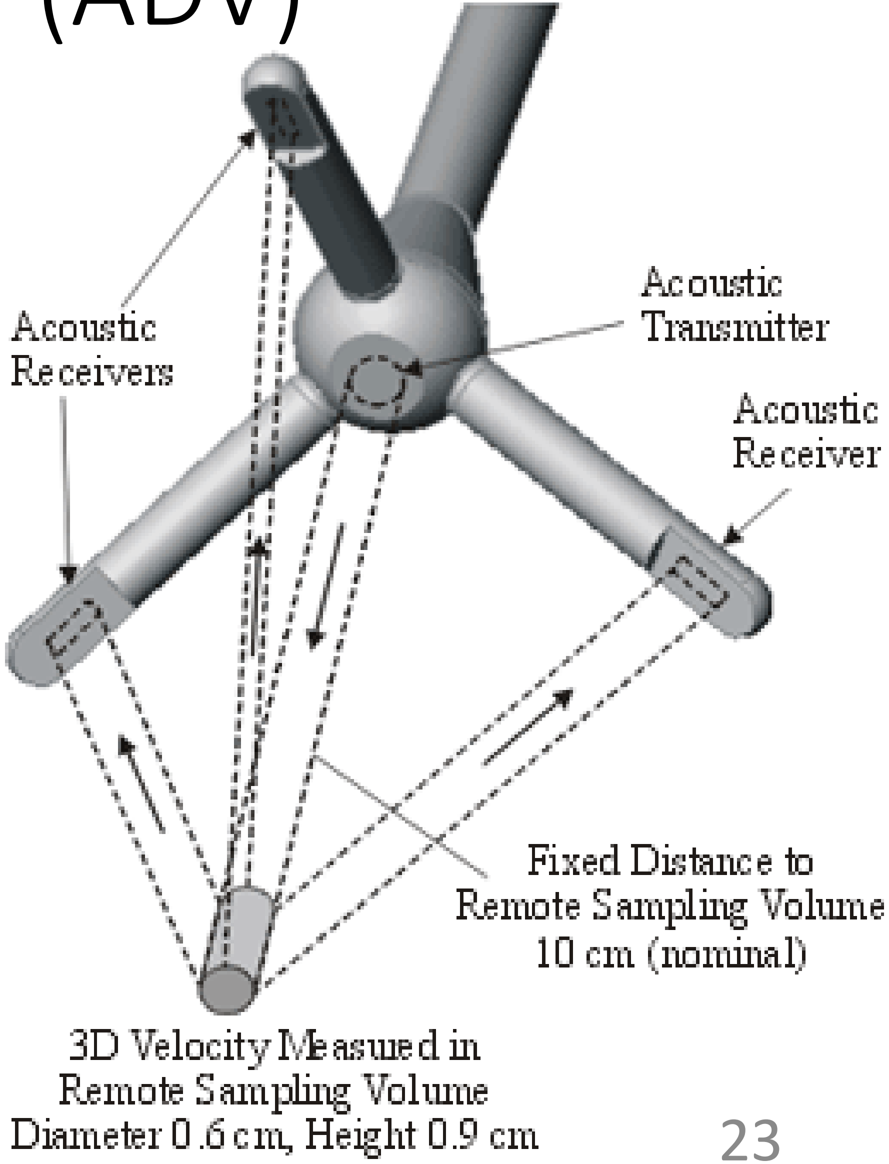



(Chevy & Raphaël, 2003;
Raphaël & de Gennes, 1996)

# Acoustic Doppler Velocimeter (ADV)






Schematic on right from SonTek.com

# Background: Surface Renewal Models

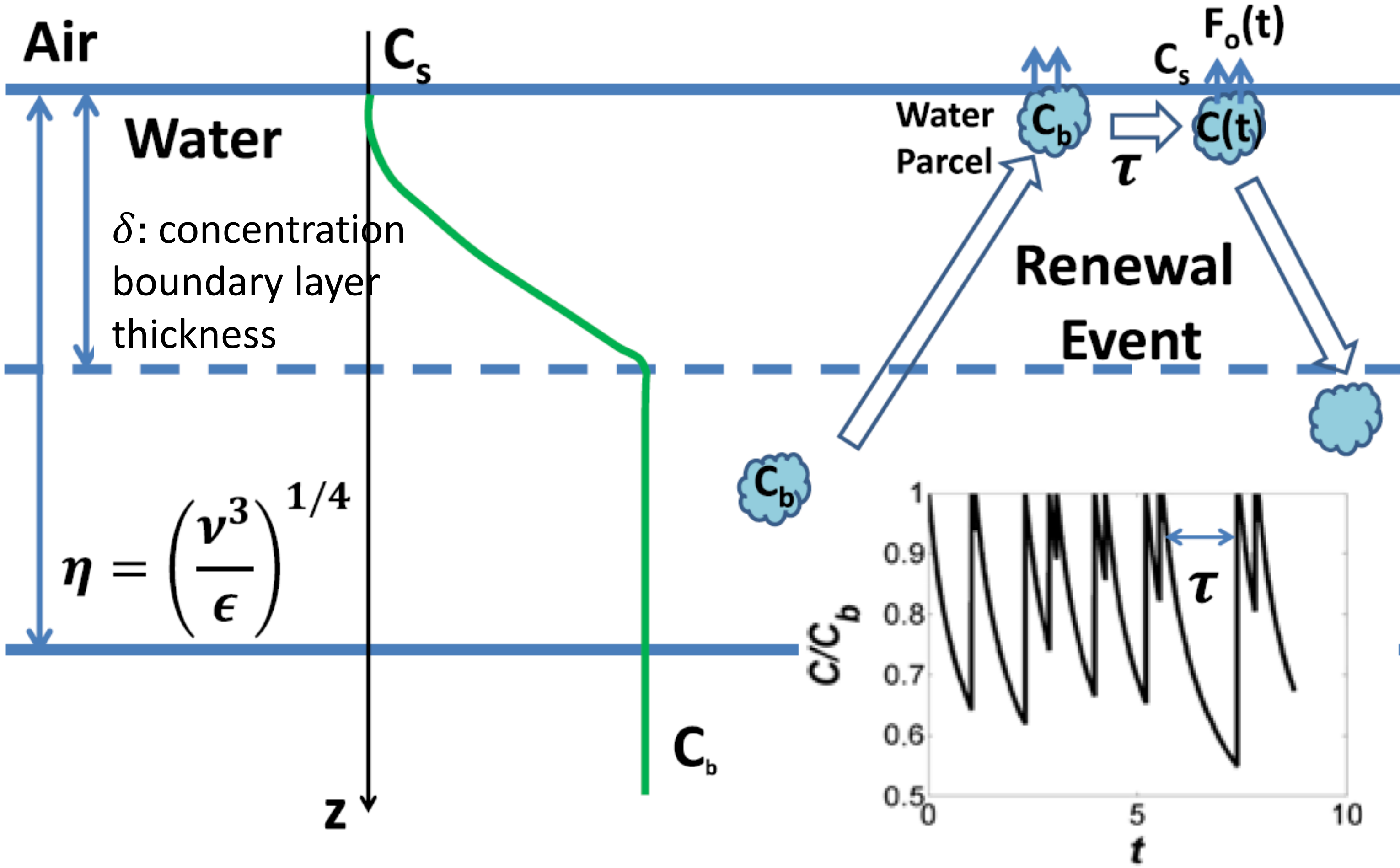
$$k \approx \sqrt{\frac{D}{\tau}}$$

$\tau$ is the average time between renewal events

For cases when the renewal time is much less than the molecular diffusion time scale:

$$\tau << T_D$$

$$T_D = \frac{\delta^2}{D}$$




(Katul and Liu, 2017)

# Background: Surface Renewal Models

(Danckwerts, 1951)

## Large Eddy Model:

$$\tau = \frac{L}{\sqrt{\overline{u'^2}}},$$

where $L$ is the turbulence integral length scale and $\sqrt{\overline{u'^2}}$ is the turbulence intensity or standard deviation of velocity, *u* (Fortescue and Pearson, 1967)

Use when $Re_t = \frac{\sqrt{\overline{u'^2}}L}{\nu} < 500$ (Theofanous *et al.*, 1975)

## Small Eddy Model: (Lamont and Scott, 1970)

$$\tau = \sqrt{\frac{\nu}{\varepsilon}} = \eta_T,$$

where $\nu$ is the kinematic viscosity and $\varepsilon$ is the energy dissipation rate (Banerjee *et al.*, 1968 and Lamont and Scott, 1970)

Use when $Re_t = \frac{\sqrt{\overline{u'^2}}L}{\nu} > 500$ (Theofanous *et al.*, 1975)

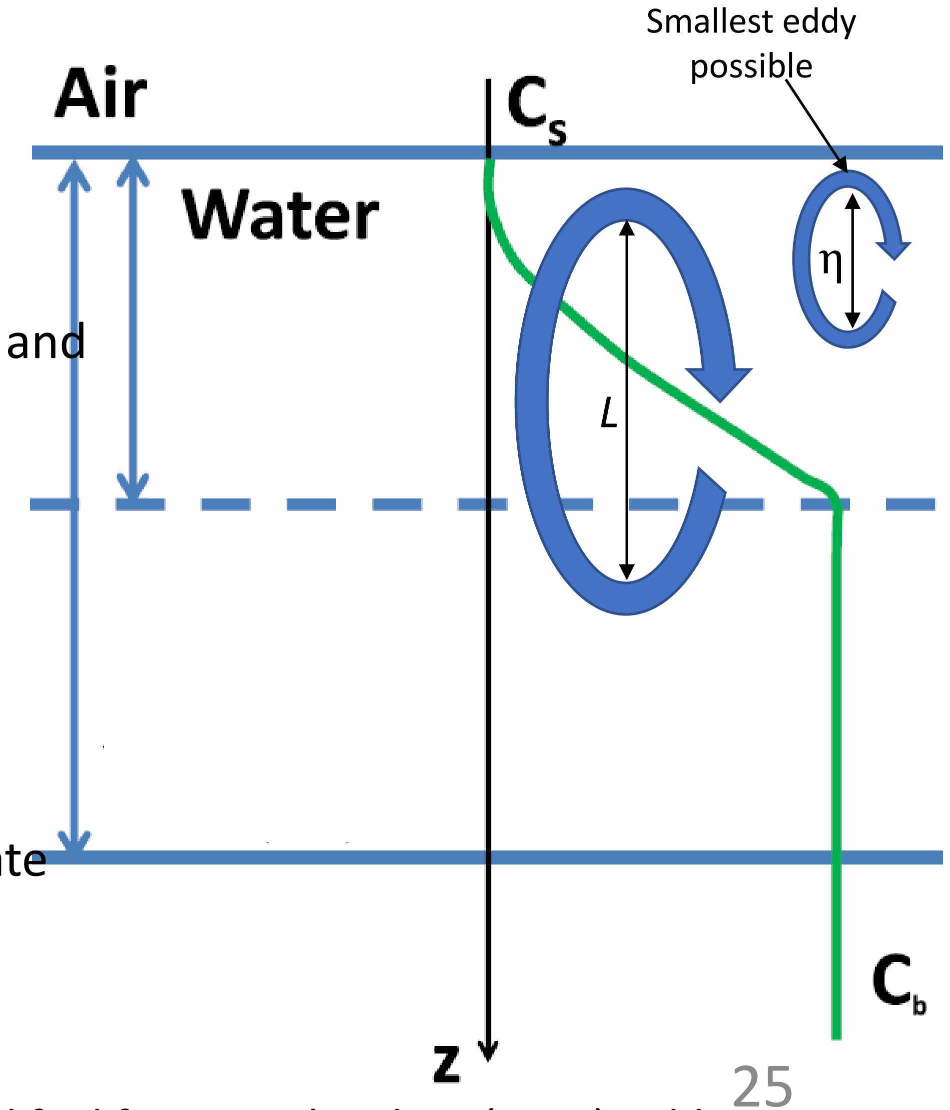
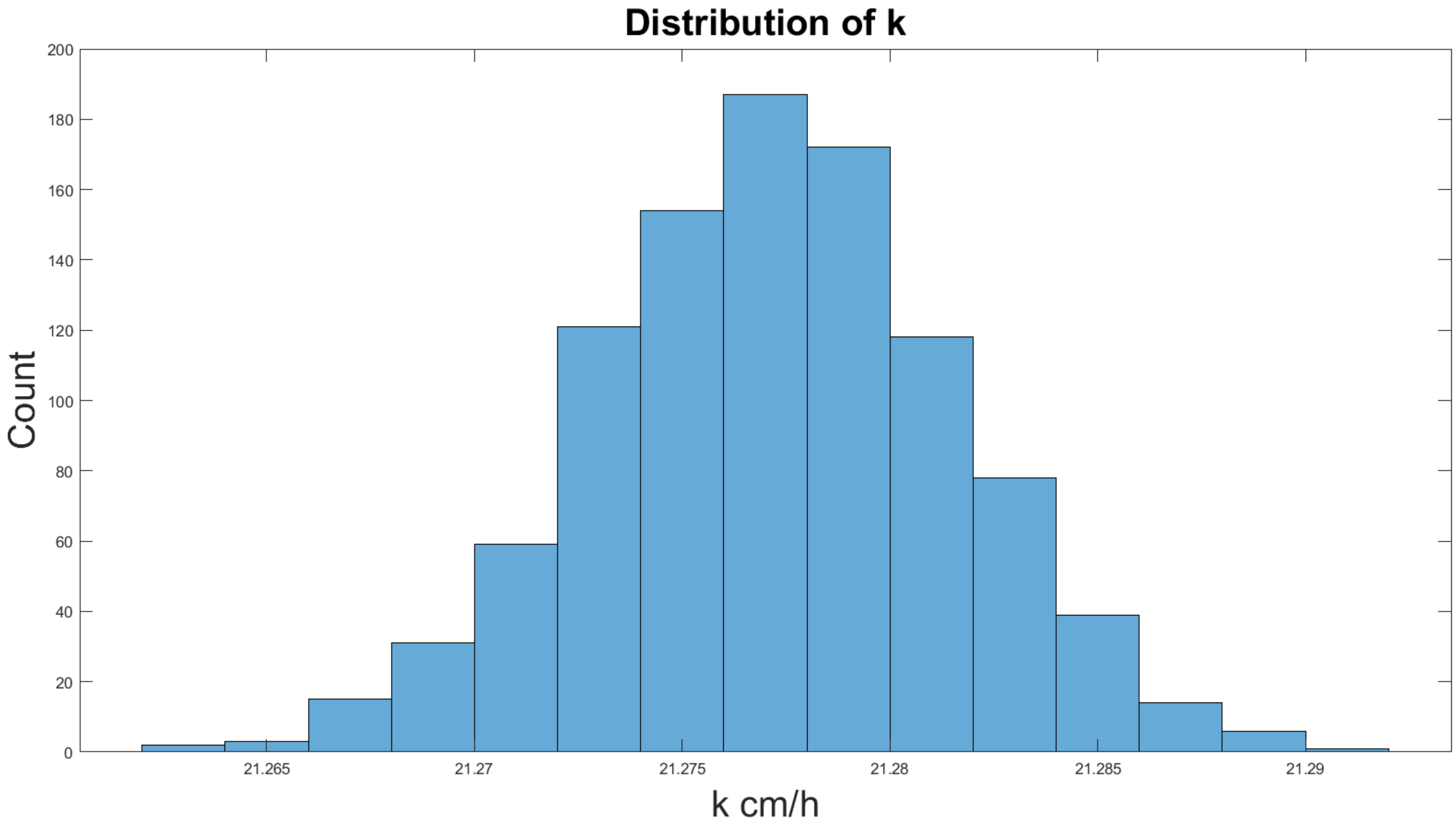





Figure modified from Katul and Liu (2017). Eddies not to scale




Resampling 1,000 times yields:

$$\bar{k} = 21.2772\,\frac{cm}{h},\ \sigma_k = 0.0044\,\frac{cm}{h},\ \mathrm{k}(975) = 21.2857\,\frac{\mathrm{cm}}{\mathrm{h}},\ \mathrm{k}(25) = 21.2684\,\frac{\mathrm{cm}}{\mathrm{h}}$$

$$\mathrm{k} = 21.277 \pm 0.009\,\frac{cm}{h}$$